Precision Tests and CP Violation in Gauge-Higgs Unification

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I. Gauge-Higgs unification (GHU)

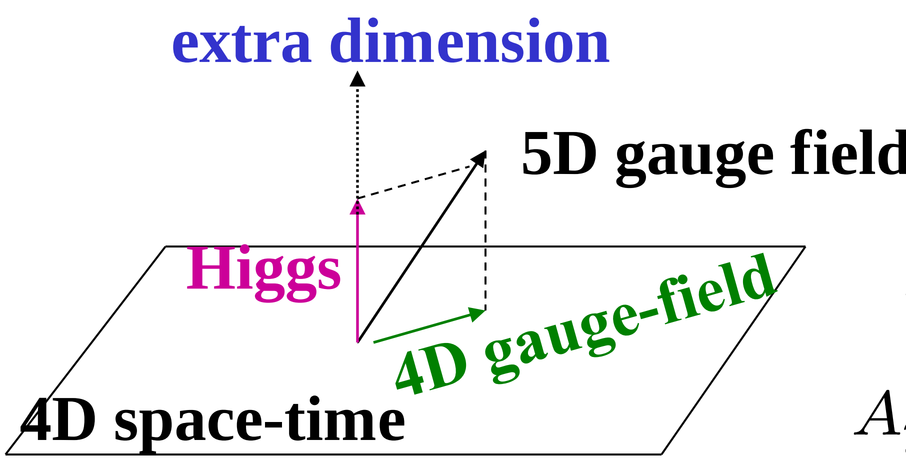
unification of gravity (s=2) & elemag (s=1) (A. Einstein)

Kaluza-Klein theory

unified theory of gauge (s=1) & Higgs (s=0) interactions

"Gauge-Higgs unification"

: realized in higher dimensional gauge theory



$$A_M = (A_\mu, A_y) \quad (5D)$$

$$A_y^{(0)}(x) = H(x) : \ \text{Higgs}$$

the idea of gauge-Higgs unification itself is not new:

- **N.S. Manton, Nucl. Phys. 58('79)141.**
- **Y. Hosotani, Phys. Lett. B126 ('83) 309 : ``Hosotani mechanism"**

$$\langle A_y^{(0)} \rangle \neq 0$$

The scenario was revived:

- **H. Hatanaka , T. Inami and C.S.L., Mod. Phys. Lett. A13('98)2601**

(the main points)

- The quantum correction to m_H is finite because of the higher dimensional gauge symmetry → **A new avenue to solve the hierarchy problem** without invoking to SUSY

- The sum over all K-K modes is essential to get the finite（for arbitrary dimensions）Higgs mass

(N.B.) The scenario may also shed some light on the arbitrariness problem in the interactions of Higgs.

II. Issues related to GHU

▪dimensional deconstruction (N. Arkani-Hamed, A.-G. Cohen, H. Georgi, Phys.Lett. B513('01)232) : latticized 5D gauge theory , @ $N \to \infty$ limit, the effective potential for H coincides with what we obtained.

▪Little Higgs model : 4D theory, where G/H of global symmetry provides Higgs as a N-G, may be "dual" to 5D GHU, where A_y associated with G/H of higher dimensional local gauge symmetry provides Higgs (holographic principle).

▪ (ultra) natural inflation (N. Arkani-Hamed, H.-C. Cheng, P. Creminelli and L. Randall, Phys.Rev.Lett. 90('03)221302; T. Inami, Y. Koyama, S. Minakami &C.S.L., Progr. Theor. Phys. (09), to appear) : $A_y^{(0)}$ may be a natural candidate for the inflaton, as the local gauge symmetry stabilizes the potential under the quantum correction

III. **Finite observables and the precision tests of GHU scenario**

For the clear test of the GHU scenario, it will be desirable to find out finite (UV insensitive) and calculable observables subject to the precision measurements, although the theory is non-renormalizable and very UV sensitive in general.

Are there such calculable observables other than the Higgs mass, protected by the higher dimensional gauge symmetry ?

Yes !

- Even in 6D , $S - (4\cos\theta_W)\, T$ is finite and predictable (w./ N. Maru , Phys. Rev.75('07)115011)
- Anomalous magnetic moment, $a = (g-2)/2$
- electric dipole moment (EDM) due to CP violation

The anomalous magnetic moment in the GHU

We consider anomalous magnetic moment of fermions, a= (g-2)/2, as a typical observable subject to the precision test.

- U(1) theory (D+1 dimensional) (Y. Adachi, C.S.L. and N. Maru, Phys.Rev. D76('07)075009)

The result is striking ! We find the anomalous moment is finite (calculable) in any space-time dimensions (in this simplified model**) in GHU scenario.**

A simple operator analysis suggests it should be the case.

In 4D, a gauge invariant operator relevant for g-2 is given as

$$\bar{\Psi}_L \sigma_{\mu\nu} \Psi_R F^{\mu\nu} \langle H \rangle + h.c. \quad (H : \text{Higgs doublet})$$

In GHU, the Higgs should be replaced by A_y and the higher dimensional gauge symmetry implies A_y appears only through covariant derivative D_y.

Thus, the relevant local gauge invariant operator should read as

$$i\bar{\Psi}\sigma_{MN}\Gamma^{A}D_{A}\Psi F^{MN}$$

Actually to get g-2, D_A should be replaced by $\langle D_A \rangle$, where $\langle A_M \rangle = \delta_M^y \langle A_y \rangle$. On the other hand, the on-shell condition for the fermion reads as

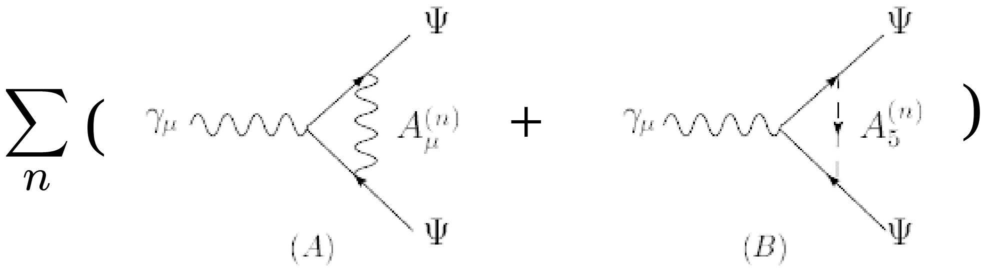
$$\Gamma^{A}\langle D_{A}\rangle\Psi = 0$$

Thus we realize that this local operator does not contribute to g-2.

Hence, we expect g-2 is observable in any space-time dimension, just as in the case of Higgs mass.

Model: D+1 dimensional "QED" on $M^D \times S^1$ (radius R)

The 1-loop diagram contributing to a = (g-2)/2.



The UV divergent and finite parts are nicely separated by use of Poisson resummation.

$$a(A)_{\mathrm{div}} = -\frac{g_D^2 \alpha M_\alpha \sqrt{\pi}}{3(4\pi)^{D/2}} \int_0^\infty ds\, s^{\frac{3-D}{2}} \quad (\alpha \equiv R M_W).$$

$$a(B)_{\mathrm{div}} = -a(A)_{\mathrm{div}} \quad \to \quad a(A)_{\mathrm{div}} + a(B)_{\mathrm{div}} = 0.$$

We find the **divergent part cancels out for arbitrary dimension D !**

The remaining finite part is given for $\alpha \equiv M_W/(1/R) \ll 1.$

$$a(D=4) \simeq \frac{5g_4^2}{16\pi^2} - \frac{7g_4^2}{288}\alpha^2.$$

The second term is the (decoupling) contribution of non-zero

K-K modes.

The first term does not reproduces the well-known result in

QED (Schwinger), $a = \frac{g_4^2}{8\pi^2},$

since the contribution of the “Higgs” A_y is comparable to that of the photon exchange, in our toy model.

・The anomalous moment in a “realistic” model: SU(3) on M^D x (S^1/Z_2) (Y. Adachi, C.S.L. and N. Maru, Phys. Rev. D79(‘09)075018; arXiv. 0904.1695 (hep-ph))

to see whether the result of Schwinger can be recovered, while keeping the cancellation of UV divergence .

(N.B.) In the GHU, gauge group should be enlarged, as the Higgs belongs to adjoint repr., while SM Higgs is SU(2) doublet.

SU(3) → SU(2) x U(1) breaking due to non-trivial Z_2- parity assignment (y: extra space coordinate) :

$$\Psi(-y) = \mathcal{P}\gamma^5\Psi(y) \quad (\mathcal{P} = \text{diag}(+,+,-))$$

Zero-modes of Gauge-Higgs sector :

$$A_\mu^{(0)} = \frac{1}{2}\begin{pmatrix} W_\mu^3 + \frac{B_\mu}{\sqrt{3}} & \sqrt{2}W_\mu^+ & 0 \\ \sqrt{2}W_\mu^- & -W_\mu^3 + \frac{B_\mu}{\sqrt{3}} & 0 \\ 0 & 0 & -\frac{2}{\sqrt{3}}B_\mu \end{pmatrix}, \qquad A_5^{(0)} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & \phi^+ \\ 0 & 0 & \phi^0 \\ \phi^- & \phi^{0*} & 0 \end{pmatrix},$$

Exactly what we need for the SU(2) x U(1) SM !

By the presence of Z_2 - odd bulk mass M,

$$\mathcal{L} = -\frac{1}{2}\mathrm{Tr}\left(F_{MN}F^{MN}\right) + \bar{\Psi}\left(i\not{D} - M\varepsilon(y)\right)\Psi$$

The Yukawa coupling and the contribution of Higgs-exchange is now strongly suppressed by a factor e^{-RM} , and the Schwinger's result is recovered (5-dimensional case).

We also get a (preliminary) lower bound on the compactification scale from the data on g-2 of 5-6 TeV :

IV. CP violation in the GHU

How to break CP symmetry is a challenging issue in the scenario of GHU, where the Higgs interactions are governed by gauge principle.

(N.B.) The low-energy limit of the open string sector of superstring theory is a sort of gauge-Higgs unification model, such as 10-dimensional (SUSY) Yang-Mills theory.
So the same problem should be addressed also in string theory.

As far as the original theory is CP invariant, possible way to break CP would be, say, ``spontaneous CP violation''.

1. CP violation due to compactification

(w./ N. Maru and K. Nishiwaki, a paper coming soon)

One of a few possibilities to break CP symmetry is to invoke to the manner of compactificaion, which determines the vacuum state of the theory.

Although the C and P transformations in higher dimensional sense can be easily found such that, for instance,

$$\psi^c = C\bar{\psi}^t, \quad C^\dagger \Gamma^M C = -(\Gamma^M)^t$$

they do not reduce to ordinary 4-dimensional transformations and should be modified.

Interestingly, the modified CP transformation is equivalent (for even space-time dimensions) to the complex conjugation of the complex homogeneous coordinates describing the pairs of extra space coordinates (a = 1 – 3, for D = 10, for instance):

$$CP: \quad z^a \ \rightarrow \ z^{a*}.$$ (C.S. Lim, Phys. Lett. B256(91)233)

Consider Type-I superstring theory with 6-dimensional Calabi-Yau manifold defined by a quintic polynomial

$$\sum_{a=1}^{5} (z^a)^5 - C(z^1 z^2 \cdots z^5) = 0$$

CP is broken only when the coefficient C is complex, since otherwise the above defining equation is invariant under $z^a \to z^{a*}$.

In fact, resultant Yukawa couplings is known to have a CP violating phase for complex C.

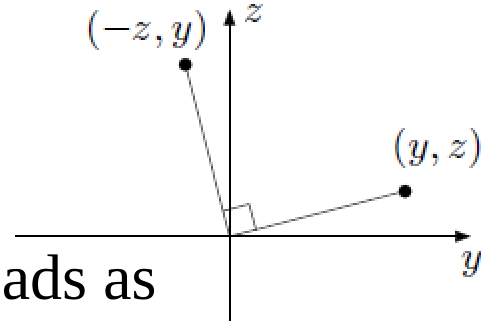
In our paper we consider much simpler compactification; we discuss the CP violation in the 6-dimensional U(1) GHU model due to the compactification on the orbifold T^2/Z_4.

We easily know that CP transfomration is not compatible with the condition of orbifolding.

In terms of a complex coordinate

$$\omega = y + iz \ (y, z : \text{extra space coordinates})$$

the orbifold condition is written as



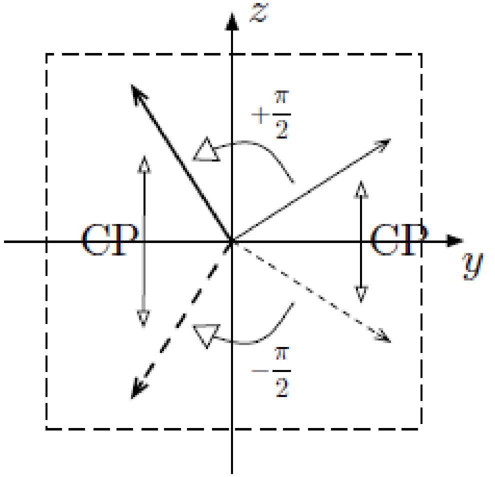
$$i\omega \sim \omega$$

After the CP transf., $\omega \to \omega^*$ the condition reads as

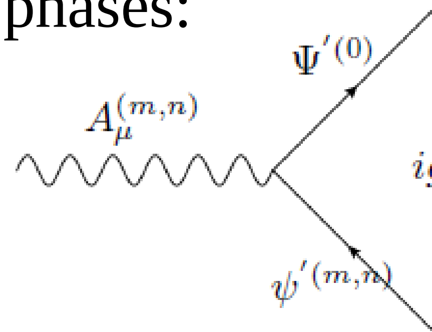
$$(-i)\omega^* \sim \omega^*$$

: “orientation-changing operator” (Strominger and Witten)

Thus, CP tranf. is not compatible with orbifolding condition , and CP symmetry is broken.



Even the interaction vertices for non-zero K-K photons generally have CP violating phases:



$$ig_4\gamma^\mu\left(\frac{1}{m_f^{(m,n)}}\frac{m+in}{R}L\right)$$

The EDM of electron, as a typical CP violating observable, however, is found to vanish at 1-loop level. Unfortunately, we anticipate that we cannot get a non-vanishing contributions even at higher loops.

In the basis of gamma matrices

$$\Gamma^{\mu} = \gamma^{\mu} \otimes 1_2, \ \Gamma^{y} = \gamma^{5} \otimes i\sigma_1, \quad \Gamma^{z} = \gamma^{5} \otimes i\sigma_2,$$

the modified P and C transf. are given as

$$P: \ \Psi_6 \ \to \ (\gamma^0 \otimes \sigma_3)\Psi_6, \ \ C: \ \Psi_6 \ \to \ (c_4 \otimes 1_2)\bar{\Psi}_6^t \ (c_4 = i\gamma_0\gamma_2).$$

Accordingly the transformation properties of vectors are uniquely determined and we find:

$$P: \ (y, z) \to (y, z) \quad C, \ CP: \ (y, z) \to (y, -z)$$

Thus P symmetry is not violated by the compactification, as is naively expected in QED.

Since, EDM necessitates both of P and CP violations, we anticipate EDM vanishes in our model, alhough we expect EDM will get contributions in a realistic theory including the SM, since P should be violated anyway in such a realistic theory.

2. CP violation due to the VEV of the Higgs

(w./ Y. Adachi and N. Maru, arXiv:0905.1022 [hep-ph] , Phys. Rev. D 80('09)055025)

Another possibility to break CP is due to the VEV of some field which has odd CP eigenvalue. We argue that the VEV $\langle A_y \rangle$ of the Higgs , or the VEV of Wilson-loop plays the role (A_y : "timeon" ?). We show that neutron EDM gets contribution already at 1-loop level in the model, though we assume the presence of only 1 generation.

(The model)

5-D SU(3) GHUmodel compactified on an orbifold S^1/Z_2 with a massive bulk fermion in a fundamental representation.

In this case, the orbifold is too simple to break CP, thus only possibility seems to be due to $\langle A_y \rangle$.

(N.B.)

To get EDM, both P and CP have to be broken. P symmetry, however, is broken anyway by the orbifolding.

In 5D CP transf. can be defined just as in the 4D case:

$$CP: \quad \Psi(x^\mu, y) \to i\gamma^0\gamma^2\Psi(x_\mu, y)^*$$

The CP transf. is known to be consistent with the orbifolding condition:

$$\Psi(-y) = \mathcal{P}\gamma^5\Psi(y) \quad (\mathcal{P} = \mathrm{diag}(+,+,-))$$

Correspondingly, the transformations of space-time and fields are fixed as.

$$CP: \ x^\mu \to x_\mu, \ y \to y, \ A_\mu(x^\mu, y) \to -A^\mu(x_\mu, y)^t, \ A_y(x^\mu, y) \to -A_y(x_\mu, y)^t.$$

Thus we realize that A_y has odd CP eigenvalue and the VEV may lead to CP violation.

Actually, when the Z_2 – odd bulk mass is switched off , we can perform a chiral rotation for Ψ, so that the coupling of A_y becomes scalar type and therefore A_y has even CP eigenvalue.
Hence, to get physical CP violating effects, the interplay between the VEV $\langle A_y \rangle$ and the bulk mass is crucial.

(The neutron EDM)

In this mechanism of CP violation, EDM appears already at 1-loop level, though we have only 1 generation.

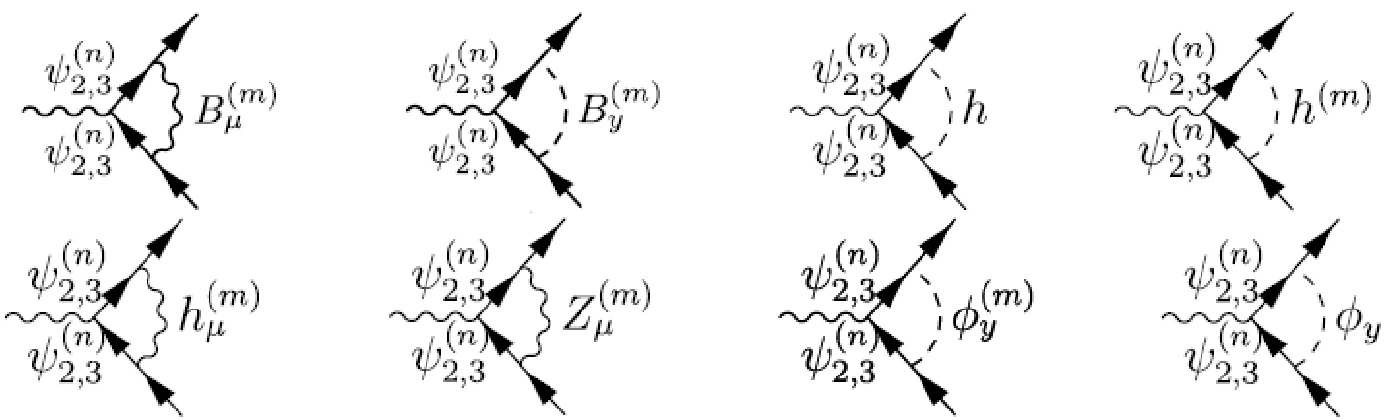


Figure 1: The diagrams contributing to EDM at one-loop by the neutral current

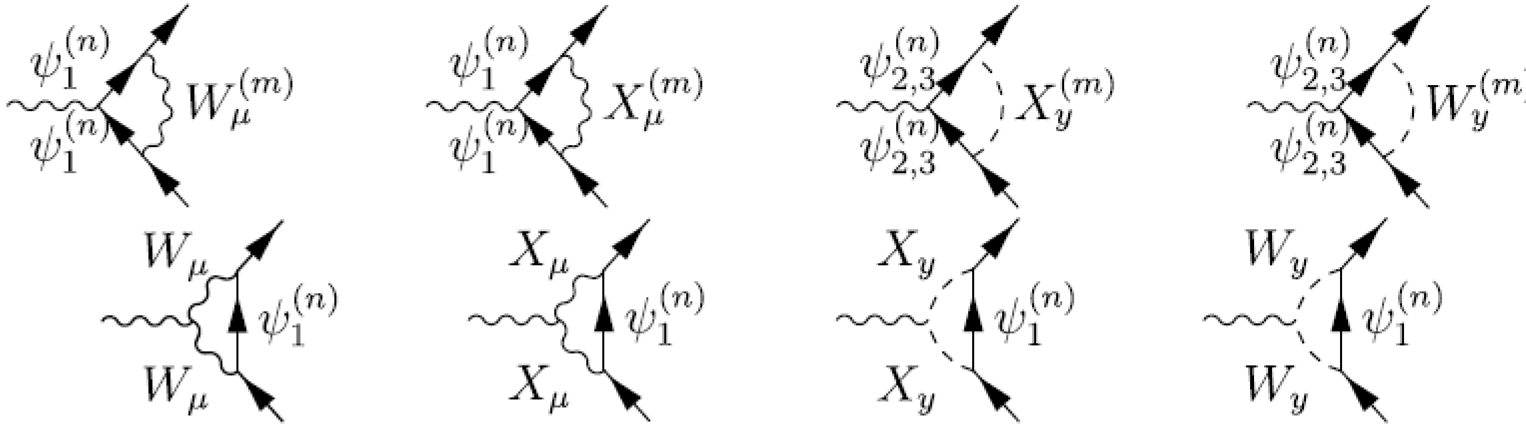


Figure 2: The diagrams contributing to EDM at one-loop by the charged current

The experimental upper bound on the EDM imposes the lower Bound on the compactification scale,

$$M_c = \frac{1}{R} \geq 2.6(\mathrm{TeV}).$$