THE MAGNETIC MOMENT OF MUON IN THE 331 ECONOMICAL MODEL AND ITS SUPERSYMMETRIC VERSION

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Chapter 1

Introduction

The Standard Model (SM) is one of the most successful models in particle physics. However, there are problems which can not be addressed in the theoretical and experiment front such as: the neutrino oscillation which indicates the very small mass of the neutrino. The neutrino mass problem is still an open question in the framework of standard model. One of the most important questions in particle physics is the question of antisymmetry between matter and antimatter. The 3 generations of quark of SM leads to the existence of CP violation in the Standard Model, however, the CP violation phase is too small to explain the current antisymmetry between matter and antimatter.

Besides, there exists theoretical questions which can not be addressed in the framework of SM such as: the question about the number of generation, hierarchy among energy scales or more specific the hierarchy between electroweak scale and the grand unify scale, the cosmological constant, the number of dimension, charge quantization.

In the last several decades, there are theoretical models introduced to explain and forecast new physics phenomenon. These models often contain new parameters therefore it is necessary to verify these models.

In 2015, The LHCb detector discovered a signal of 3.6$\sigma$ deviation of B meson decay. And most recently, ATLAS report show a 2.6$\sigma$ in the decay channel $\tilde{\chi}_2^0 \rightarrow \tilde{\chi}_1^0 Z \rightarrow \tilde{\chi}_1^0 l^+l^-$. These signals show that high energy physics is at stage of new discovery.

1.1 Research objective

Investigate the muon anomalous magnetic moment in extend models. More specifically:

- Investigate the contribution to the muon magnetic moment in E331 model and SUSYE331 model.
Investigate the parameter space constrained from muon anomalous magnetic moment experiment data in the 331 model and SUSYE331 model.

Investigate mass spectrum of Higgs with even and odd CP in SUSYE331 model.

1.2 Research objects

One loop contribution to the muon anomalous magnetic moment in the economical 331 model (E331).

One loop contribution to the muon anomalous magnetic moment in the supersymmetric economical 331 model (SUSYE331).

Higgs potential and mass spectrum in the SUSYE331 model.

1.3 Research contents

E331 model, SUSYE331 model.

Full form of Higgs potential in the SUSYE331.

Charged Higgs and neutral Higgs mass spectrum with odd and even CP.

Parameter space of E331 model and SUSYE331 model.

1.4 Methods

Quantum field theory.

Mathematica software for numerical calculation.

1.5 Structure of thesis

Besides the introduction, conclusions and a section dedicated to the publication, the content of the thesis is presented in two chapters.

Chapter 2, general introduction to E331 model. Contribution to muon magnetic moment in E331 model.

Chapter 3, general introduction to SUSYE331 model. Investigate the full form of the SUSYE331 model. Investigate the muon magnetic moment in SUSYE331 model. Comparing with experiment data to constrain the mass of the supersymmetric particle of the model.
Chapter 2

Muon anomalous magnetic moment in E331 model

2.1 Review of 3-3-1 model

The class of 331 models based on the extending the gauge group $SU(2)_L \to SU(3)_L$ has more Higgs field compared to standard model. Anomaly cancellation leads to the number of triplet equals the number of antitriplet. Unlike the standard model in which the anomaly is cancelled in each generation, in 331 model the anomaly is cancelled when 3 generations are taken into account.

Since the 331 model is extended from the standard model therefore it includes the fermion structure of the standard model. More specifically, the two upper parts of the fermion triplet are identified with that of standard model. The third part of the triplet are identified with heavy quark or new lepton depending on the triplet. The minimal choice is to introduce the conjugate of the charged lepton in the third component of lepton triplet.

The charge operator of 331 model is the combination of diagonal Gell-mann matrix:

$$\hat{Q} = \hat{T}^3 + \beta \hat{T}^8 + X \mathbb{1}.$$  \hfill (2.1)

The parameter $\beta$ is important and model dependence.

The extension of the gauge group leading to the appearance of new gauge boson such as the neutral gauge boson $Z'$ and the charged boson $V'^Q$. The new neutral $Z'$ leads to the existence of flavour changing neutral current(FCNC) at tree level.

2.2 E331 model

E331 model has parameter $\beta = - \frac{1}{\sqrt{3}}$. Compared with other 331 models, the E331 model has some intriguing features such as the E331 model has only two Higgs triplet and can explain the small mass of neutrino.
2.3 Contribution to muon anomalous magnetic moment in E331

The new contribution to the muon anomalous magnetic moment at one loop level are charged boson $Y^\pm$, neutral boson $Z'$, neutral scalar $H^0_1$ and charged scalar $H^+$. 

2.3.1 Charged vector boson

The contribution of charged vector boson is:

$$\Delta a_{V,A}^\mu (V^\pm) = \frac{(f_V)^2}{8\pi^2} \frac{m_\mu^2}{m_V^2} \int_0^1 dx \frac{F_{V,A}(x)}{\epsilon^2 \lambda^2 (1-x)(1-\epsilon^{-2}x) + x},$$

(2.2)

In E331 model, $m_\nu, m_{\nu c} \ll m_\mu \ll m_V$ therefore we can approximate $\epsilon, \lambda \to 0$.

$$\Delta a_\mu (Y^\pm) = \left[ (f_V)^2 + (f_A)^2 \right] \left( \frac{m_\mu^2}{m_Y^2} \right) \left( \frac{5}{3} \right).$$

(2.3)

The contribution of W boson in Standard model is:

$$a_{W}^{SM} (W) = \sqrt{2} G_\mu m_\mu \frac{10}{3^2} \times 388.70(0) \times 10^{-11}$$

(2.4)

with $G_\mu = \frac{g^2 \sqrt{2}}{8 m_W^2}$.

Like W boson, the contribution to $a_\mu$ of $Y$ boson is:

$$\Delta a_\mu (Y^\pm) = \frac{g^2 m_\mu}{32 \pi^2 m_Y^2} \left( \frac{10}{6} \right) = a_{W}^{SM} (W) \times \frac{m_W^2}{m_Y^2}.$$  

(2.5)

The electroweak contribution of $W$ in (2.4) is at the same order with the contribution of the new particle $\Delta a_{\mu}^{NP}$. This show that the mass of the new charged boson is at the same order with the mass of $W$ boson.

2.3.2 Contribution of new neutral boson

The contribution of the new neutral boson is given as:

$$\Delta a_\mu (Z') = \frac{1}{8\pi^2} \frac{m_\mu^2}{m_Z^2} \int_0^1 dx \frac{\left( C_{V}' \right)^2 F_V(x) + \left( C_{A}' \right)^2 F_A(x)}{(1-x)(1-\lambda^2 x) + \epsilon^2 \lambda^2 x},$$

(2.6)
\[ \Delta a_\mu(Z'(f)) = a_\mu^{SM}(Z(f)) \times \frac{m_Z^2}{m_{Z'}^2} \left[ -\frac{c_\varphi}{\sqrt{3 - 4s_W^2}} + s_\varphi \right]^2, \quad (2.7) \]

Where \( a_\mu^{SM}(Z(f)) = -193.89(2) \times 10^{-11} \) is the electroweak contribution of Z boson in the Standard Model, \( \varphi \) is the mixing angle between Z and \( Z' \).

### 2.3.3 Neutral scalar

\[ \Delta a_\mu(h^0) = \frac{f_{h^0}^2}{8\pi^2} \frac{m_\mu^2}{m_{h^0}^2} \int_0^1 dx \frac{x^2(1 + \epsilon - x)}{(1 - x)(1 - \lambda^2 x) + \epsilon^2\lambda^2 x}, \quad (2.8) \]

where \( \epsilon = 1, \lambda = m_\mu/m_{h^0} \).

Contribution of new Higgs \( H_1^0 \) of E331 model is:

\[ \Delta a_\mu^{E331}(H_1^0) = \frac{\sqrt{2}G_\mu m_\mu^4}{64\pi^2} \frac{m_W^2}{(m_Y^2 + m_W^2)} \frac{1}{m_{H_1^0}^2} \left[ \ln \left( \frac{m_{H_1^0}}{m_\mu} \right) - \frac{7}{12} \right], \quad (2.9) \]

### 2.3.4 Charged scalar

\[ \Delta a_\mu(h^\pm) = \frac{f_{h^\pm}^2}{8\pi^2} \frac{m_\mu^2}{m_{h^\pm}^2} \int_0^1 dx \frac{-x(1 - x)(x + \epsilon)}{\epsilon^2\lambda^2(1 - x)(1 - \epsilon^{-2} x) + x}, \]

where \( \epsilon = m_{\nu, e}/m_\mu, \lambda = m_\mu/m_{h^\pm} \).

The total contribution of charged Higgs is:

\[ \Delta a_\mu^{E331}(h^\pm) = -\left( \frac{1}{6} \right) \frac{\sqrt{2}G_\mu m_\mu^2}{4\pi^2} \times \frac{m_{H_2^+}^2}{m_{H_2^+}^2 + m_Y^2 + m_W^2}, \quad (2.10) \]

where \( m_{H_2^+}^2 = \frac{2\lambda_4}{g} m_Y^2 \).

The contribution of neutral scalar and charged scalar cancel out each other therefore the main contribution to muon magnetic moment \( \Delta a_\mu^{NP} \) in E331 model is:

\[ \Delta a_\mu^{NP} = a_\mu(Y^+) + a_\mu(Z'(f)). \quad (2.11) \]

The contribution to \( a_\mu^{NP} \) in (2.11) depends on \( m_Y^+ \) hence \( a_\mu^{NP} \) can give constraints on value of \( m_Y \) or the symmetry breaking scale \( SU(3)_L \). These value are show on figure 2.1.

We can see that the total contribution to muon magnetic moment in the model E331 is not big enough to explain the experiment data of \( a_\mu \).
2.3.5 Symmetry breaking condition in some 331 models

Reduced 331 model (R331)

The contribution to $\Delta a_{\mu}^{NP}$ in R331 model is given as followings:

- $V^+$ gauge boson:
  \[
  \Delta a_{\mu}(V^+) = a_{\mu}^{SM}(W) \times \frac{m_W^2}{m_{V^+}^2}.
  \] (2.12)

- The contribution of $Z'$ boson can be written as:
  \[
  \Delta a_{\mu}^{R331}(Z') = -a_{\mu}^{SM}(W) \times \frac{m_Z^2}{m_{Z'}^2} \times \frac{3(1-4s_W^2)}{20}.
  \] (2.13)

- Contribution of $U^{++}$ boson:
  \[
  a_{\mu}^{R331}(U^{++}) = a_{\mu}^{SM}(W) \times \frac{42}{5} \times \frac{m_W^2}{m_{U^{++}}^2}.
  \] (2.14)

The total contribution to $a_{\mu}$ in R331 model, $\Delta a_{\mu}^{R331} = a_{\mu}(V^+) + a_{\mu}^{R331}(Z') + a_{\mu}^{R331}(U^{++})$, depends on $m_{V^+}$ which is characterized by $SU(3)_L$ breaking scale. The numerical results are given in figure 2.2.

3-3-1 model with heavy exotic lepton

The 331 model with heavy exotic lepton can give large contribution to $a_{\mu}$.

\[
\Delta a_{\mu}(V^+) \equiv a_{\mu}^{SM}(W) \frac{3m_W^2}{10m_{V^+}^2} f(k),
\] (2.15)

with $k = m_{N_R}^2/m_{V^+}^2$. $f(k)$ is investigated as in Fig. 2.3
Figure 2.2: Contribution of gauge boson to $a_\mu$ in RM331 model.

Figure 2.3: Plot $f(k)$ against $k = \frac{m_{N_R}^2}{m_{V^+}^2}$ in two case: $k < 1$ (left) and $k > 1$ (right).

We can see that $f(k)$ is of order $O(1)$ hence $\Delta a_\mu(V^+)$ is smaller than $a_\mu^{SM}(W)$ by a factor $\frac{3m_W^2}{10m_{V^+}^2}$. $m_{V^+}^2$ is therefore of order with $m_W^2$ when $\Delta a_\mu(V^+)$ is the same order with $\Delta a_\mu^{NP}$. 

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Chapter 3

Muon anomalous magnetic moment and Higgs potential in supersymmetric E331 model

3.1 Supersymmetry

The theoretical motivations of supersymmetry are: grand unify: in supersymmetry the three coupling constants of electroweak, electromagnetic, and strong interaction are converge. In standard model, these coupling constants do not converge at any scale. However, in the minimal supersymmetric standard model these coupling constants converge; Dark matter: in supersymmetric model the lightest supersymmetric particle neutralino is the candidate of dark matter; Hierarchy problem, supersymmetric give natural explanation to the hierarchy problem.

3.2 The SUSYE331 model

3.2.1 Particle content of the SUSYE331 model

Superfields are defined as followings:

\[ \hat{F} = (\tilde{F}, F), \quad \hat{S} = (S, \tilde{S}), \quad \hat{V} = (\lambda, V), \quad (3.1) \]

Where \( F, S \) and \( V \) are fermion, scalar and vector field and their super partner are denoted as follows: \( \tilde{F}, \tilde{S} \) and \( \lambda \)

The fermionic superfield are given as:

\[ \hat{L}_{aL} = (\hat{\nu}_a, \hat{\tilde{l}}_a, \hat{\nu}^c_a)^T_L \sim (1, 3, -1/3), \quad \hat{\nu}^c_{aL} \sim (1, 1, 1), \quad (3.2) \]

\[ \hat{Q}_{1L} = (\hat{u}_1, \hat{d}_1, \hat{u'}_1)^T_L \sim (3, 3, 1/3), \]
\[ \tilde{u}_{1L}^c, \tilde{u}_{2L}^c \sim (3^*, 1, -2/3), \quad \tilde{d}_{1L}^c \sim (3^*, 1, 1/3), \]

\[ \tilde{Q}_{\alpha L} = \left( \tilde{d}_{\alpha}, -\tilde{u}_{\alpha}, \tilde{d}_{\alpha}^c \right)_L^T \sim (3^*, 3^*, 0), \quad \alpha = 2, 3, \]

\[ \tilde{u}_{\alpha L} \sim (3^*, 1, -2/3), \quad \tilde{d}_{\alpha L} \sim (3^*, 1, 1/3), \]

The number in side parentheses denote quantum number of symmetry group \((SU(3)_C, SU(3)_L, U(1)_X)\). \(\tilde{\nu}_L = (\tilde{\nu}_R)^c\) and \(a = 1, 2, 3\) is generation index.

### 3.2.2 Neutralino sector

The neutralino mass term are given as follows:

\[ \mathcal{L} = \left( \tilde{\psi}^o \right)^\dagger M_N \tilde{\psi}^o, \tag{3.3} \]

where

\[ \tilde{\psi}^o = \left( \tilde{\chi}_1^o, \tilde{\chi}_1^o, \tilde{\chi}_2^o, \tilde{\chi}_2^o, \tilde{\rho}_1^o, \tilde{\rho}_1^o, \lambda_B, \lambda_3, \lambda_8, \right. \]

\[ \lambda_X = \frac{1}{2}, \lambda_X^* = \frac{-1}{2}. \]

In general, we can find a basis to diagonalize the mass matrix above meaning there exits an unitary matrix \(U\) satisfy

\[ UM_N U^\dagger = \text{Diag}(m_{\tilde{\chi}_1}^2, m_{\tilde{\chi}_2}^2, m_{\tilde{\chi}_3}^2, m_{\tilde{\chi}_4}^2, m_{\tilde{\rho}_1}^2, m_{\tilde{\rho}_2}^2, m_{\lambda_3}^2, m_{\lambda_8}^2). \tag{3.4} \]

### 3.2.3 Chargino sector

In this section we will investigate the mass term of Higgsinos and charged gauginos. There are four charged gauginos and six charged Higgsinos. In the basis:

\[ \psi^+ = (\tilde{W}^+, \tilde{\nu}_1^+, \tilde{\rho}_1^+, \tilde{\rho}_2^+, \tilde{\chi}_1^+), \psi^- = (\tilde{W}^-, \tilde{\nu}_1^-, \tilde{\rho}_1^-, \tilde{\rho}_2^-, \tilde{\chi}_1^-). \tag{3.5} \]

The Lagrangian describe the mass term of chargino are given as:

\[ \mathcal{L}_{\text{charginomass}} = \left( \tilde{\psi}^\pm \right)^\dagger M_{\psi} \tilde{\psi}^\pm + H.c, \tag{3.6} \]

with \(M_{\psi}\)

\[ M_{\psi} = \begin{pmatrix} 0 & \mathcal{M} \\ \mathcal{M}^T & 0 \end{pmatrix}, \tag{3.7} \]
where $\mathcal{M}$ is $5 \times 5$ given by

$$
\mathcal{M} = 
\begin{pmatrix}
M_{YW} & 0 & \frac{g_{\nu'}}{\sqrt{2}} & 0 & \frac{g_u}{\sqrt{2}} \\
0 & M_{Y} & 0 & \frac{g_{\nu'}}{\sqrt{2}} & \frac{g_w}{\sqrt{2}} \\
\frac{g_{\nu}}{\sqrt{2}} & 0 & \mu_{\rho} & 0 & 0 \\
0 & \frac{g_{\nu'}}{\sqrt{2}} & 0 & \mu_{\rho} & 0 \\
\frac{g_{u'}}{\sqrt{2}} & \frac{g_{w'}}{\sqrt{2}} & 0 & 0 & \mu_{\chi}
\end{pmatrix}.
$$

(3.8)

3.2.4 Lepton mixing

The lepton flavour violation (LFV) are now gaining interests. The lepton mixing is observed in the mixing of $\nu_\tau, \nu_\mu$. In fact, there exists mixing in all three generations. However, the mixing in the second and the third is largest. In this thesis, we assume that there exists mixing in slepton sector more precisely the mixing of left and right $\tilde{\mu}$.

3.2.5 Smuon and sneutrino mass.

The super potential gives relevant contribution to $(g - 2)_\mu$ are given as follows:

$$
W' = \mu_{0a} \hat{L}_{aL} \hat{\chi}^\prime + \mu_{\chi} \hat{\chi} \hat{\chi}^\prime + \mu_{\rho} \hat{\rho} \hat{\rho}^\prime + \gamma_{ab} \hat{L}_{aL} \hat{\rho}^\prime \tilde{l}_{bL}
+ \lambda_a \epsilon \hat{L}_{aL} \hat{\chi} \hat{\rho} + \lambda_{ab} \epsilon \hat{L}_{aL} \hat{\rho} \hat{L}_{bL} \hat{\rho},
$$

(3.9)

with $\mu_{0a}, \mu_{\rho}$ and $\mu_{\chi}$ has mass dimension one, other coefficients in $W'$ are dimensionless and $\lambda_{ab} = -\lambda'_{ba}$. The softly breaking term are given by:

$$
-\mathcal{L}_{SMT} = M_{ab}^2 \tilde{L}_{aL} \tilde{L}_{bL} + m_{ab}^2 \tilde{\nu}_{aL} \tilde{\nu}_{bL} + \left\{ M_{a}^2 \chi^\dagger \tilde{L}_{aL} + \eta_{ab} \tilde{L}_{aL} \rho \tilde{\rho}_{bL} + v_{a} \epsilon \tilde{L}_{aL} \rho + \varv_{ab} \epsilon \tilde{L}_{aL} \tilde{L}_{bL} \rho + \omega_{a} \epsilon \tilde{Q}_{aL} \tilde{d}_{c}^L
+ \omega'_{ab} \tilde{L}_{aL} \tilde{Q}_{aL} \tilde{d}_{c}^L + H.c. \right\},
$$

(3.10)

where $\varv_{ab} = -\varv_{ba}$. This Lagrangian are responsible for sfermion mass by combining D term and F term.

3.3 Scalar potential for Higgs sector

There are connections among magnetic moment, electric moment and CP violation. The 331 model has many source of CP violation. In this part we will investigate the CP odd Higgs and CP even Higgs and their mass spectrum.
The full form of the Higgs potential of model SUSYE331 is given as follow:

\[
V_{\text{SUSYE331}} \equiv V_{\text{scalar}} + V_{\text{soft}} = \mu^2 \chi^4 (\chi^\dagger \chi + \chi'^\dagger \chi') + \mu^2 \rho^4 (\rho^\dagger \rho + \rho'^\dagger \rho') + \frac{g'^2}{12} \left( -\frac{1}{3} \chi^\dagger \chi + \frac{1}{3} \chi'^\dagger \chi' + \frac{2}{3} \rho^\dagger \rho - \frac{2}{3} \rho'^\dagger \rho' \right)^2 + \frac{g^2}{8} \sum_{b=1}^{8} (\chi_i^b \lambda_{ij}^b \chi_j - \chi_i^b \lambda_{ij}^b \chi_j' + \rho_i^b \lambda_{ij}^b \rho_j - \rho_i^b \lambda_{ij}^b \rho_j')^2 + m^2_\rho \rho^\dagger \rho + m^2_\chi \chi^\dagger \chi + m^2_{\rho'} \rho'^\dagger \rho' + m^2_{\chi'} \chi'^\dagger \chi' - (b_\rho \rho \rho' + b_\chi \chi \chi' + \text{H.c.}). \tag{3.11}
\]

Expand the Higgs around VEVs and solve for the minimum of the potential and using notation as follows:

\[
\tan \beta = t_\beta = \frac{u}{w'}, \quad \tan \gamma = t_\gamma = \frac{v}{w'}, \quad t = g'/g, \tag{3.12}
\]

we have:

\[
\cos 2\gamma \text{ and } \cos 2\beta \text{ can be determined through soft parameters as:}
\]

\[
c_{2\gamma} \equiv \cos 2\gamma = \frac{2c^2_W \left( \frac{1}{4} \mu_\rho^2 + m^2_\rho - \frac{b_\rho}{t_\gamma} \right) + \left( \frac{1}{4} \mu_\chi^2 + m^2_\chi - \frac{b_\chi}{t_\beta} \right)}{m^2_W},
\]

\[
c_{2\beta} \equiv \cos 2\beta = \frac{\left( \frac{1}{4} \mu_\rho^2 + m^2_\rho - \frac{b_\rho}{t_\gamma} \right) + 2 \left( \frac{1}{4} \mu_\chi^2 + m^2_\chi - \frac{b_\chi}{t_\beta} \right)}{m^2_\chi}. \tag{3.13}
\]

Since \(|c_{2\gamma}|, |c_{2\beta}| \leq 1\) therefore:

\[
\left| \frac{1}{4} \mu_\rho^2 + m^2_\rho - \frac{b_\rho}{t_\gamma} \right| \sim \left| \frac{1}{4} \mu_\chi^2 + m^2_\chi - \frac{b_\chi}{t_\beta} \right| \sim O(m^2_W), \tag{3.14}
\]

\[
\left| \frac{1}{4} \mu_\rho^2 + m^2_\rho - \frac{b_\rho}{t_\gamma} \right| \sim \left| \frac{1}{4} \mu_\chi^2 + m^2_\chi - \frac{b_\chi}{t_\beta} \right| \sim O(m^2_\chi). \tag{3.15}
\]

In the next part we will investigate Higgs mass spectrum by using approximation \(\epsilon = m^2_W/m^2_\chi, \epsilon \ll 1\). We will first determine the mass of pseudo-scalar Higgs and then use this as parameter in finding Higgs mass spectrum.

### 3.3.1 CP odd Higgs

The mass term in Lagrangian of pseudo Higgs is split into two parts.

\[
\mathcal{L}_A^{\text{mass}} = \frac{1}{2} (A_1, A_2, A_3, A4) \times M^2_A (A_1, A_2, A_3, A4)^T + \frac{1}{2} (A_5, A_6) M^2_{A_\rho} (A_5, A_6)^T. \tag{3.16}
\]
Diagonalizing these matrix give us 3 non-zero eigenvalues and 3 zero eigenvalue corresponding to 3 eigenstate with mass and 3 massless eigenstate.

\[
m^2_{A1} = m^2_{H_{A1}} = \frac{2b_\rho}{s_2} = \frac{1}{2}\mu_\rho + m_\rho + m_{\bar{\rho}},
\]

\[
m^2_{A2} = m^2_{H_{A2}} = \frac{2b_\chi}{s_2} = \frac{1}{2}\mu_\chi + m_\chi + m_{\bar{\chi}},
\]

\[
m^2_{A3} = m^2_{H_{A3}} = m^2_{A2} + m^2_{\chi}.
\]

Three massive eigenstate can be rewritten as follows:

\[
H_{A1} = A_5 c_\gamma + A_6 s_\gamma,
\]

\[
H_{A2} = A_1 c_\beta s_\zeta + A_2 c_\beta c_\zeta + A_3 s_\beta s_\zeta + A_4 s_\beta c_\zeta,
\]

\[
H_{A3} = -A_1 c_\beta c_\zeta + A_2 c_\beta s_\zeta - A_3 s_\beta c_\zeta + A_4 s_\beta s_\zeta,
\]

where \( \tan \zeta = u'/w', \cos \zeta = c_\zeta, \sin \zeta = s_\zeta, \cos \beta = c_\beta, \sin \beta = s_\beta, \cos \gamma = c_\gamma, \sin \gamma = s_\gamma. \) Three massless eigenstate are:

\[
H_{A4} = -A_5 s_\gamma + A_6 c_\gamma,
\]

\[
H_{A5} = -A_2 s_\beta + A_4 c_\beta,
\]

\[
H_{A6} = -A_1 s_\beta + A_3 c_\beta.
\]

### 3.3.2 Neutral Scalar Higgs

In the basis \((S_1, S_2, S_3, S_4, S_5, S_6)\) square mass of CP even Higgs satisfies: \( \det (M^2_{6S} - \lambda I_6) = 0, \)

\[
\lambda \left[ \lambda - (1 + \frac{t_\beta^2}{2}) \left( \frac{b_\chi}{t_\beta} + \frac{g^2}{4}(u'^2 + w'^2) \right) \right] f(\lambda) = 0, \quad (3.20)
\]

\[
f(\lambda) = a\lambda^4 + b\lambda^3 + c\lambda^2 + d\lambda + e. \quad (3.21)
\]

We can define and approximate \( \epsilon \)

\[
\epsilon \simeq \frac{m_W^2}{m_{z'}^2} \times \frac{4c_W^2}{4c_W^2 - 1}, \quad (3.22)
\]

Denote \( \lambda = X \times m^2_X, f(\lambda) = 0 \) can be written in the form

\[
g(X) = AX^4 + BX^3 + CX^2 + DX + E = 0, \quad (3.23)
\]

where

\[
A = 1,
\]

\[
B = - \left( 4c_W^2 h_W^2 + k_1 + k_2 + 4h_W^2 \times \epsilon \right),
\]

\[
C = 4c_W h_W^2 k_1 k_2 c_{\beta\gamma} + k_1 k_2 + h_W^2 \left( 1 + k_1 c_{\beta\gamma} + k_2 \right) \times \epsilon,
\]

\[
D = -4c_W h_W^2 k_1 k_2 c_{\beta\gamma} - 4h_W^2 \left[ k_2 c_{\beta\gamma} + c_{\gamma\gamma} k_1 (1 + k_2) \right] \times \epsilon,
\]

\[
E = 4h_W^2 k_1 k_2 c_{\gamma\gamma} c_{\beta\gamma} \times \epsilon. \quad (3.24)
\]
3.3.3 Charged Higgs

In the basis \((\chi^+, \chi'^+, \rho^+_1, \rho^+_2, \rho'^+_1, \rho'^+_2)\), the square mass of charged Higgs are solutions of the equation

\[
\text{Det}(M^2_{\text{charged}} - \lambda I_6) = 0.
\]

(3.25)

By changing variables like the case of neutral Higgs we have:

\[
X^2 \left[ \lambda - (m^2_{A_1} + m^2_W) \right] \times f(X) = 0,
\]

(3.26)

with \(X = m^2_{H^\pm}/m^2_X\) where \(m_W\) is the mass of \(W\) boson. The function \(f(X)\) has the form:

\[
f(X) = X^3 + AX^2 + BX + C,
\]

(3.27)

where

\[
A = -(1 + k_1 + k_2 + \epsilon), \quad B = -c^2_{2\beta} + k_1(1 + c_2\gamma c_{2\beta} + k_2)
\]

\[
+ [k_2 + c_2\gamma c_{2\beta}(2 + k_2)] \times \epsilon - c^2_{2\gamma} \times \epsilon^2,
\]

\[
C = (1 + \epsilon) \left[ c_{2\beta} - c_{2\gamma}(\epsilon + k_1) \right] \left[ c_{2\beta}(1 + k_2) - c_{2\gamma}\epsilon \right].
\]

(3.28)

3.3.4 Constraints of Higgs Masses

In this part we will investigate in detail the Higgs mass spectrum. We will investigate the soft parameters in Electroweak scale and \(SU(3)_L\) scale.

Soft-parameters in the electroweak breaking scale

Solve for Higgs mass spectrum in this case we have the mass of one light Higgs at order \(O(GeV)\) and two other light Higgses

\[
m^2_{H^0_{2,3}} \simeq \frac{1}{2} \left( m^2_Z + m^2_{A_1} + \sqrt{(m^2_{A_1} - m^2_Z)^2 + 4s^2_{2\gamma}m^2_Zm^2_{A_1}} \right).
\]

(3.29)

This formula (3.29) has the same form with the formula represent the neutral Higgs in the model MSSM. At tree level the Higgs mass is smaller then \(m_Z|c_{2\gamma}|\). If we indentify this Higgs with 125.5 GeV Higgs "like" discovered recently at LHC then we need a large correction since at tree level this Higgs mass is smaller then \(m_Z\) therefore the parameter space in this case is restricted. Hence, the symmetry breaking scale should not be at \(SU(2)_L\) scale. In the next part we will investigate the \(SU(3)_L\) symmetry breaking scale.
Soft-parameters in the SU(3) scale

CP-even neutral Higgses

Like the case the soft parameters in the $SU(2)_L$ scale. Solving for the Higgs mass spectrum we have:

$$m_{H^0_{3,4}}^2 = \frac{1}{2} \left( m_{A_2}^2 + m_{Z'}^2 \right) \pm \sqrt{\left( m_{A_2}^2 - m_{Z'}^2 \right)^2 + 4m_{Z'}^2 m_{A_2}^2 s^2_{2\beta}} ,$$

(3.30)

Where $m_{Z'}^2 = 4m_X^2 c_W^2 / (4c_W^2 - 1)$. Like the case $SU(2)_L$, at tree level, the mass of light Higgs in (3.30) is smaller than $m_{Z'} c_{2\beta}$. In order to indentify light Higgs with 125.5 GeV Higgs, in this case we need a small correction since $m_{Z'}$ is at $SU(3)_L$ scale which is at order TeV. By choosing the suitable value of $c_{2\beta}$ will give us the larger value of Higgs mass.

![Figure 3.1: Plot mass of $m_{H^0_j}$ ($j = 1, 2, ..., 5$) as function of $m_{A_1}$](image1.png)

![Figure 3.2: Mass of lightest neutral Higgs including top quark and stop quark correction.](image2.png)
In figure 3.1 we compare analytical result with numerical result. We can see that the four blue curve represent four heavy Higgs while lightest Higgs has mass \( m_{H^0} \approx m_Z \) when \( t_\gamma \gg 1 \). The mass of lightest Higgs can be reach experiment bound if corrections are included. In model SUSYE331 we can point out that lightest Higgs can have mass correction like the Higgs in model MSSM. In figure 3.2, we investigate the mass of lightest neutral Higgs as function of top quark and stop quark correction. It can bee seen that the mass of the lightest Higgs can reach 125-126 GeV with \( M_X = 2 \text{TeV} \).

### Charged Higgs

Solving the charged Higgs mass spectrum we have:

\[
m^2_{H^\pm_1} = m_X^2 + m_{A_2}^2,
\]

\[
m^2_{H^\pm_{2,3}} = \frac{1}{2} \left( m_{A_1}^2 \mp \sqrt{m_{A_1}^2 - 2m_X^2 c_2 \beta c_2 \gamma} \right)^2 + 4m_X^2 c_2 \beta s_2 \gamma^2,
\]

(3.32)

Impose the positive value condition on the mass of Higgs in (3.32) we have

\[
c_2 \beta \left( c_2 \beta - k_1 c_2 \gamma \right) < 0.
\]

(3.33)

equivalently

\[
\frac{m_{A_1}^2 + m_W^2}{m_X^2} c_2 \gamma < c_2 \beta < \frac{c_2 \gamma m_W^2}{m_X^2 + m_{A_2}^2} < 0.
\]

(3.34)

If this condition is satisfied then the masses of charged Higgses in model SUSYE331 are at \( SU(3)_L \) scale.

![Figure 3.3: Plot \( m^2_{H^\pm} \) as function of \( m_{A_1} \).](image)

We investigate the mass of charged Higgs like the case of neutral Higgs. Figure 3.3 correspond to the case \( t_\gamma \) and \( t_\beta \) large and \( c_2 \gamma \simeq c_2 \beta = -1 \). Inserting these values into (3.32) we have two value \( m^2_{H^\pm} = \{ m_X^2, m_{A_1}^2 - m_X^2 \} \) meaning in order to eliminate tachyon Higgs then \( m_{A_1} \) have value greater than \( m_X \).
3.4 SUSY contributions to muon MDM

The magnetic moment $a_\mu$ can be evaluated in weak eigenstate where the dependence on supersymmetric parameters are more reveal than in mass eigenstate.

3.4.1 Weak eigenstate

The contribution to $a_\mu$ can be divided into 5 parts as followings:

$$a_{\mu}^{SUSYE331} = a_{\mu L}^{(a)} + a_{\mu R}^{(a)} + a_{\mu L}^{(b)} + a_{\mu R}^{(b)} + a_{\mu LR}^{(c)}$$  \hspace{1cm} (3.35)

3.4.2 Numerical Investigation

we can classified the parameters of the model SUSYE331 as followings:

- $B/\mu$ term: $\mu_\rho$, ratio of two vacua $\tan \gamma$
- Gauginos mass $m_{\lambda_B}, m_{\lambda_A}$
- Right handed slepton mass: $m_{\tilde{l}_R^2}, m_{\tilde{\nu}_R^2}, m_{\tilde{l}_R^3}, m_{\tilde{\nu}_R^3}$
- Left handed slepton mass: $m_{\tilde{l}_L^2}, m_{\tilde{\nu}_L^2}, m_{\tilde{l}_L^3}, m_{\tilde{\nu}_L^3}$
- Mixing terms : $A_\mu, A_\tau, A_{\tau_\mu}^L, A_{\tau_\mu}^R$.

Our goal is to point out the dependence of muon MDM on $\tan \gamma, \mu_\rho$ and $m_\mu^2/m_{SUSY}^2$. By using experiment results we can find the upper bound of the light supersymmetric particle in model SUSYE331.

To simplify our calculation we can take approximation by taking the mass of all particle to the SUSY scale $m_{SUSY}$. The mixing terms are non-diagonal elements of mixing matrix therefore small and can be eliminated. In this limit we have:

$$\Delta a_{\mu \text{total}} = -\frac{1}{36} \frac{g^2}{8\pi^2} \frac{m_\mu^2}{m_{SUSY}^2} + \frac{1}{108} \frac{g'^2}{8\pi^2} \frac{m_\mu^2}{m_{SUSY}^2} + \frac{4}{9} \frac{g^2}{8\pi^2} \text{sgn}(\mu_\rho) \times \frac{\tan \gamma}{m_{SUSY}^2} + \frac{1}{648} \frac{g'^2}{8\pi^2} \text{sgn}(\mu_\rho) \tan \gamma \frac{m_\mu^2}{m_{SUSY}^2}.$$  \hspace{1cm} (3.36)

Where $\text{sgn}(\mu_\rho)$ is sign function of $\mu_\rho$.

From (3.36) we can conclude that $\Delta a_\mu$ is the same sign with $\text{sgn}(\mu_\rho)$. In 3.4, we compare muon MDM with experiment results. we evaluate the value of muon MDM with different values of $\tan \gamma = 1, 5, 10, 20, 40, 60$. To explain the $\geq 3.6\sigma$ deviation between experiment results and what predicted by theoretical model the mass of supersymmetric particle can be $\approx 75$ GeV if $\tan \gamma = 1$. 

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When $\tan \gamma$ large, ($\tan \gamma=60$), the mass of supersymmetric particle, $m_{SUSY}$, can be 900 GeV to address $2\sigma$ deviation.

Next we will fix the value of $\tan \gamma$, the mass of the second generation sleptons are assumed to have the same mass $m\tilde{l}_2$ and the mass of the third generation slepton have the same mass $m\tilde{l}_3$. The hierarchy among masses of the second and third generation is considered. The mixing is maximal meaning $\theta_L = \theta_R = \frac{\pi}{4}, \theta_{\nu L} = \theta_{\nu R} = \frac{\pi}{4}$.

Since model MSSM is the effective model of model SUSY E331 therefore we can use the constraints on the mass of smuon by experiment in which the mass of smuon is greater than 91 GeV. The mass of the second generation can be approximated as $m\tilde{l}_2 = 100$ GeV and $m\tilde{l}_2 = 200$ GeV respectively while the mass of the third generation is $m\tilde{l}_3 = 1$ TeV. The contributions of supersymmetry on muon magnetic moment are illustrated in figure 3.5 inwhich $\tan \gamma = 5$. The mass of bino and gauginos are assumed to have the same mass. In figure 3.5a, muon MDM is investigated with $m\tilde{l}_2 = 100$ GeV, $m\tilde{l}_3 = 1$ TeV while in figure 3.5b, $m\tilde{l}_2 = 200$ GeV, $m\tilde{l}_3 = 1$ TeV. The asymmetry in figure 3.5a, 3.5b caused by terms which do not depend on $\mu_\rho$. From 3.5a we can see that in order to explain the muon anomalous magnetic moment the mass of gauginos is about $200 \leq m_G \leq 700$ GeV. And because of the way we chose the mass, $\tan \gamma = 5$ is the smallest value to address $2 - 3.6 \sigma$ deviation. We can also see that the value of magnetic moment is inverse proportional to the value of
\( m_{\tilde{\ell}} = 500 \text{ GeV} \) \hspace{1cm} (a)

\( m_{\tilde{\ell}} = 800 \text{ GeV} \) \hspace{1cm} (b)

Figure 3.6: Investigare MDM as function of \( m_G \) and \( \mu_\rho \)

\( m_{\lambda_A} = 1 \text{ TeV} \) \hspace{1cm} (a)

\( m_{\lambda_A} = 2 \text{ TeV} \) \hspace{1cm} (b)

Figure 3.7: Muon MDM as function of left-handed slepton mass \( m_{\tilde{\ell}_L} \) and right-handed slepton mass \( m_{\tilde{\ell}_R} \)

\( \mu_\rho \). In the case the mass of second generation is 200 GeV as in figure 3.5b, the value of MDM is enough to explain 2.0 \( \sigma \) deviation. From this we can find the upper bound 200 GeV of the mass of the second generation while the mass of the third generation is 1 TeV and \( \tan \gamma = 5 \).

Figures 3.5a and 3.5b show that the contribution to muon MDM in the model SUSYE331 is improved the small region of \( m_G \).

By choosing the maximal value of \( \tan \gamma \) we can find the upper or lower bound of the mass of supersymmetric particle. In this part we will investigate with \( \tan \gamma = 60 \) which is the maximal value currently investigated at (ACHARD 04), we will perform numerical calculation the contribution of SUSYE331 in this case. The hierarchy between mass of second and third generation is the same as above. Figures 3.6a, and 3.6b we investigate muon MDM as function of \( m_G \) and \( \mu_\rho \) with condition as above except the mass of the second generation is chosen to be 500 GeV fig. 3.6a and 800 GeV fig. 3.6b while the mass of the third generation is 2 TeV. Result as fig. 3.6a show that the upper bound of the mass of gauginos is \( m_G = 1500 \text{ GeV} \) and \( \mu_\rho = 1500 \text{ GeV} \). However fig. 3.6b show that the upper bound of \( m_G = 1100 \text{ GeV} \) and \( \mu_\rho = 1200 \text{ GeV} \).

Next, the mass of Bino is fix and equals to 350 GeV, left handed sleptons of second and third generation have the same mass \( m_{\tilde{\ell}_L} \), right handed slepton of
the second and third generation have the same mass $m_{	ilde{R}_2}$, $\tan \gamma = 60$, $\mu_{\rho} = 140$ GeV, $m_{\lambda_A} = 1$ TeV (fig. 3.7a) and 2 TeV (fig. 3.7b). With $m_{\lambda_A} = 1$ TeV, results as in figure 3.7a give the lower bound of $m_{\tilde{L}} \simeq 400$ GeV and there is no upper bound of $m_{\tilde{R}}$. Figure 3.7b show that the upper bound of left handed slepton is $m_{\tilde{L}} \leq 800$ GeV. We use the upper bound of the left-handed slepton of third generation 800 GeV and fix $\tan \gamma = 60$, $\mu_{\rho} = 140$ GeV, $m_{\lambda_B} = 350$ GeV, the contribution to muon MDM is investigated as function of second generation left-handed slepton and second generation right-handed slepton. $m_{\tilde{l}_{L2}} = m_{\tilde{\nu}_{L2}} = m_{\tilde{L}_2}$, $m_{\tilde{l}_{R2}} = m_{\tilde{\nu}_{R2}} = m_{\tilde{R}_2}$, the result as in figures 3.8a and 3.8b. Compare with experiment we can find the lower bound of $m_{\tilde{L}_2} > 400$ GeV with $m_{\lambda_A} = 1$ TeV and the upper bound of left-handed slepton of second generation is 600 GeV with $m_{\lambda_A} = 2$ TeV. However there is no upper bound for second generation right handed slepton.

At last, we will investigate in specific case where $\theta_R = \theta_L = 0$ and $\theta_{\nu_R} = \theta_{\nu_L} = \frac{\pi}{4}$ meaning there is no mixing in slepton sector while the mixing in sneutrino sector is maximal. In figures 3.9a and 3.9b, the muon MDM is in-
vestigated as function of $m_{\tilde{\nu}_L^2}$ and $m_{\tilde{R}_2}$ with upper bound of $m_{\tilde{L}_2} = 600$ GeV from above constraints while other parameters is fixed: $\tan \gamma = 60$, $\mu_\rho = 140$ GeV, mass of third generation of slepton is 800 GeV, $m_{\lambda_B} = 350$ GeV, $m_{\lambda_A} = 1$ TeV (left) and $m_{\lambda_A} = 2$ TeV (right). The mass of sneutrino is constrained to be smaller than 550 GeV to explain 2 $\sigma$ deviation.
Chapter 4

CONCLUSION

In this thesis we investigate in detail the muon anomalous magnetic moment in model E331 and model SUSYE331. In the case of model E331: we derive the analytical expression of muon MDM; numerical investigate contributions to muon MDM; compare analytical results and experiment data.

This investigation leads to conclusions: muon anomalous magnetic moment is proportional to $\frac{m_\mu^2}{M_{NP}^2}$. Like the case of standard model, gauge bosons give main contribution to muon MDM. Since the coupling constant of scalar field is very small therefore the contribution of scalar field can be ignored. By comparing with experiment leads to conclusion: The model E331 does not give fully explanation to recent experiment data. This conclusion leads to the investigation of muon MDM in model SUSYE331.

Before investigate muon MDM in SUSYE331 we will analyze the full form of super potential in model SUSYE331. By adding the soft term $b/\mu$, vacua is stabilized and tachyon Higgs is eliminated. Solving for the minimum of the potential we point out soft term and $b/\mu$ term in this model are in $SU(3)_L$ scale. In the limit of large value of soft parameters, $b/\mu$ terms, $t_\gamma$ and $t_\beta$, the Higgs mass spectrum of model SUSYE331 resemble many characteristic of Higgses of MSSM model.

Muon MDM is then investigated in detail in model SUSYE331. We calculate the one-loop contribution and numerical investigation of muon MDM in SUSYE331 model in weak eigenstate. In our calculation we have assumed that: The mixing in slepton and sneutrino is maximal: mixing terms $A_\tau$, $A_\mu$ and $A_{L,R}^{\mu\tau}$ are small and can be ignored. Especially in the limit where all masses equal to $m_{SUSY}$ we obtain the reduced analytical formula of muon MDM in SUSYE331 model. This formula shows that the contribution to muon MDM in model SUSYE331 is improved in the region of small value of $m_{SUSY}$ and large value of $\tan\gamma$. The numerical investigation show that in order to explain experiment data, $m_{SUSY} \approx 75$ GeV with $\tan\gamma = 1$ and $m_{SUSY} = 900$ with $\tan\gamma = 60$. In the case where $\tan\gamma$ is small, ($\tan\gamma = 5$), the mass of one gen-
eration of slepton is at order $\mathcal{O}(1)$ TeV. The mass of light slepton is bounded in the range $[100, 200 GeV]$ and $\Delta a^\mu$ can reach the current bound. When $\tan \gamma$ is large ($\tan \gamma = 60$), the mass of left handed slepton is bounded to be 800 GeV. If we only consider the maximal mixing in the sneutrino sector then the upper bound of sneutrino will be 550 GeV. These values can be verify at Large Hadron Collider (LHC) or in the next generation of accelerator such as ILC.
LIST OF PUBLICATIONS


Main results used in this thesis are published in publication 1 and 3