Large Mixing of Light and Heavy Neutrinos in Seesaw Models

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Introduction

* Neutrinos are now known experimentally to have mass and mix.
* Among various ways beyond the standard model (SM) proposed to explain this, the most popular are the seesaw scenarios: new particles with sufficiently large masses are introduced to make the light-neutrino masses small.
* In seesaw models of types I and III, the new heavy particles are fermions – the heavy neutrinos & charged leptons.
* The seesaw mechanism would be best tested by searching for these heavy particles, hopefully at the LHC.
* Their production and detectability depends on the strength of their interactions with SM particles, particularly on the mixing between the light and heavy neutrinos.

Type-I seesaw

* The seesaw mechanism involves right-handed neutrinos N_R that are singlets under the SM gauge groups and hence can have large Majorana masses. Minkowski, Yanagida, Gell-Mann et al., Glashow
* The mass Lagrangian for light neutrinos ν_L and heavy neutrinos N_R

$$\mathcal{L} = -\overline{N_R}\, Y_D \tilde{H}^\dagger L_L - \tfrac{1}{2}\overline{N_R}\, M_N (N_R)^c + \text{H.c.}$$

Y_D is the Yukawa coupling, $H = \left(\phi^+ \;\; \phi^0\right)^{\rm T} = \left(\phi^+ \;\; (v+h+i\eta)/\sqrt{2}\right)^{\rm T}$ the Higgs doublet with vacuum expectation value v, $\tilde{H} = i\tau_2 H^*$,
$L_L = \left(\nu_L \;\; l_L^-\right)^{\rm T}$ the left-handed lepton doublet,
M_N the Majorana mass of N_R, and $(N_R)^c$ its charge conjugate.

* The resulting mass terms for the neutrinos

$$\mathcal{L}_{\rm mass} = -\tfrac{1}{2}\left(\overline{(\nu_L)^c} \quad \overline{N_R}\right) M_{\rm seesaw} \begin{pmatrix} \nu_L \\ (N_R)^c \end{pmatrix} + \text{H.c.}$$

the seesaw mass matrix $\quad M_{\rm seesaw} = \begin{pmatrix} 0 & m_D^{\rm T} \\ m_D & M_N \end{pmatrix}$

and the Dirac mass $\quad m_D = v Y_D/\sqrt{2}$

Neutrino masses in type-I seesaw

* In terms of the mass eigenstates ν_{mL} and N_{mL}

$$\begin{pmatrix} \nu_L \\ (N_R)^c \end{pmatrix} = U \begin{pmatrix} \nu_{mL} \\ N_{mL} \end{pmatrix}, \qquad U \equiv \begin{pmatrix} U_{\nu\nu} & U_{\nu N} \\ U_{N\nu} & U_{NN} \end{pmatrix}$$

U is unitary and diagonalizes $M_{\rm seesaw}$,

$$\begin{pmatrix} \hat{m}_\nu & 0 \\ 0 & \hat{M}_N \end{pmatrix} = U^{\rm T} M_{\rm seesaw} U$$

* Assuming $m_D \ll M_N$ and expanding in $m_D M_N^{-1}$, one finds to leading order

$$U_{\nu N} = m_D^\dagger \hat{M}_N^{-1} , \qquad U_{N\nu} = -M_N^{-1} m_D U_{\nu\nu} , \qquad U_{NN} = 1 ,$$

$m_\nu \equiv -m_D^\dagger \hat{M}_N^{-1} m_D^*$, which is diagonalized by the unitary Pontecorvo-Maki-Nakagawa-Sakata matrix $U_{\rm PMNS}$

$$\hat{m}_\nu = U_{\rm PMNS}^\dagger m_\nu U_{\rm PMNS}^* ,$$

$U_{\nu\nu}$ has small deviations from unitarity $\Rightarrow$ take $U_{\nu\nu} = U_{\rm PMNS}$

Neutrino interactions in type-I seesaw

* In terms of the weak eigenstates, the neutrinos couple to the gauge and Higgs bosons in the SM according to

$$\mathcal{L}' = \left(\frac{g}{\sqrt{2}} \overline{l_L} \gamma^\mu \nu_L W_\mu^- - \overline{N_R} m_D \nu_L \frac{h}{v} + \text{H.c.} \right) + \frac{g}{2c_\text{w}} \bar{\nu}_L \gamma^\mu \nu_L Z_\mu$$

$c_\text{w} = \cos\theta_\text{W}$

* In the mass-eigenstate basis,

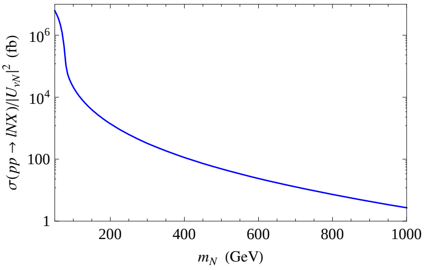
$$\begin{aligned} \mathcal{L}' \;=\; & \frac{g}{\sqrt{2}} \Big(\overline{l_L} \gamma^\mu U_{\nu\nu} \nu_{mL} W_\mu^- + \overline{l_L} \gamma^\mu U_{\nu N} N_{mL} W_\mu^- + \text{H.c} \Big) \\ & + \frac{g}{2c_\text{w}} \Big(\overline{\nu_{mL}} \gamma^\mu U_{\nu\nu}^\dagger U_{\nu\nu} \nu_{mL} + \overline{N_{mL}} \gamma^\mu U_{\nu N}^\dagger U_{\nu\nu} \nu_{mL} \\ & \qquad + \overline{\nu_{mL}} \gamma^\mu U_{\nu\nu}^\dagger U_{\nu N} N_{mL} + \overline{N_{mL}} \gamma^\mu U_{\nu N}^\dagger U_{\nu N} N_{mL} \Big) Z_\mu \\ & - \Big[\overline{(\nu_{mL})^c} \hat{m}_\nu U_{\nu\nu}^\dagger U_{\nu\nu} \nu_{mL} + \overline{(N_{mL})^c} \hat{M}_N U_{\nu N}^\dagger U_{\nu\nu} \nu_{mL} \\ & \qquad + \overline{(\nu_{mL})^c} \hat{m}_\nu U_{\nu\nu}^\dagger U_{\nu N} N_{mL} + \overline{(N_{mL})^c} \hat{M}_N U_{\nu N}^\dagger U_{\nu N} N_{mL} + \text{H.c} \Big] \frac{h}{v} \end{aligned}$$

* Through mixing, N can interact with the SM gauge bosons.

Heavy-neutrino production in type-I seesaw

* N can then be singly produced via $q\bar{q}' \to W^* \to lN$ and $q\bar{q} \to (h^*, Z^*) \to \nu N$, the former with a charged lepton l in the final state being easier to observe.
* The LHC can, in principle, test the seesaw mechanism for m_N values up to a TeV or so.
* All these N-production processes depend on $U_{\nu N}$ $\Rightarrow$ its size is crucial for the testability of the model.
* It is interesting to see how large $U_{\nu N}$ can be, taking into account constraints from existing data.

$pp \to lNX$ due to $q\bar{q}' \to W^* \to lN$



Light-heavy mixing in type-I seesaw with one generation

* For 1 generation of light and heavy neutrinos, the light neutrino mass

$$m_\nu \simeq -\frac{m_D^2}{M_N}\,, \qquad m_D \ll M_N$$

* Light-heavy mixing $U_{\nu N}$ has size $|m_D/M_N| = \sqrt{|m_\nu/M_N|}$
* Since experimentally $m_\nu \lesssim \mathcal{O}(1\,\text{eV})$,

$$|U_{\nu N}| \;<\; 10^{-5}\sqrt{100\,\text{GeV}/|M_N|}$$

* This mixing is tiny, even with M_N as low as 100 GeV.
* This makes it not possible to produce enough number of heavy neutrinos to study them at the LHC.

Light-heavy mixing in type-I seesaw with more than 1 generation

- This turns out to be not necessarily true in general.
- It is possible to have much larger $U_{\nu N}$ with more than 1 generation:

$$U_{\nu N} = m_D^\dagger \, \hat{M}_N^{-1} \,, \qquad m_\nu \equiv -m_D^\dagger \, \hat{M}_N^{-1} \, m_D^* \,, \qquad \hat{m}_\nu = U_{\rm PMNS}^\dagger \, m_\nu \, U_{\rm PMNS}^*$$

$$\Rightarrow \quad U_{\rm PMNS} \, \hat{m}_\nu \, U_{\rm PMNS}^{\rm T} = -U_{\nu N} \, \hat{M}_N \, U_{\nu N}^{\rm T}$$

- If there's a nontrivial solution for $U_{\nu N}$ which makes the right-hand side vanish, the elements of $U_{\nu N}$ can be large and evade the constrain $|U_{\nu N}|^2 = |m_\nu / m_N|$ in the 1-generation case.

Buchmuller et al., Ingelman & Rathsman, Gluza, Pilaftsis, Kersten & Smirnov, Ma

- Denote such a solution by U_0: $\quad U_0 \, \hat{M}_N \, U_0^{\rm T} = 0$
- Once U_0 is found, one can add a perturbation U_δ to U_0, so that $U_{\nu N} = U_0 + U_\delta$ leads to the light-neutrino masses and mixing.

Rank of U_0

* In the basis where U_0 is diagonalized, M_N generally is not diagonal.
* Since M_N must be of rank 3 in order that the 3 heavy neutrinos have nonzero masses, the rank of U_0 must be ≤ 2, for otherwise $\det(U_0 M_N U_0^{\rm T}) \neq 0$, contradicting the assumption $U_0 M_N U_0^{\rm T} = 0$
* If U_0 has rank 2, one can write its diagonal form as $\hat{U}_0 = {\rm diag}(a,b,0)$ leading to

$$\hat{U}_0 M_N \hat{U}_0^{\rm T} = \begin{pmatrix} a^2 M_{11} & ab\, M_{12} & 0 \\ ab\, M_{12} & b^2 M_{22} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

  M_{ij} are the elements in M_N, which is symmetric.
* Since M_N is of rank 3, $M_{12,11,22}$ cannot all be simultaneously zero.
* If at least one of a and b is kept nonzero, then only 2 nontrivial solutions are possible: (1) $b=0,\ M_{11}=0$ and (2) $a=0,\ M_{22}=0$
* We conclude that U_0 must be a rank-1 matrix.

General form of U_0 with rank 1 satisfying $U_0 \hat{M}_N U_0^{\rm T} = 0$

* The diagonal form of U_0 can be $\hat{U}_0 = {\rm diag}(\hat{u},0,0)$ with $\hat{u}$ being a constant.
* This is related to the nondiagonal U_0 by the biunitary transformation $U_0 = V'\hat{U}_0 V$ where V and V' are 3×3 unitary matrices.
* Denoting the elements of $V^{(\prime)}$ by $V_{kl}^{(\prime)}$, we then arrive at

$$U_0 = \kappa \begin{pmatrix} a\,V_{11} & a\,V_{12} & a\,V_{13} \\ b\,V_{11} & b\,V_{12} & b\,V_{13} \\ c\,V_{11} & c\,V_{12} & c\,V_{13} \end{pmatrix},$$

  κ is a constant, $\kappa a = \hat{u} V'_{11}$, $\kappa b = \hat{u} V'_{21}$, and $\kappa c = \hat{u} V'_{31}$
* Using $U_0 = V'\hat{U}_0 V$ in $U_0 \hat{M}_N U_0^{\rm T} = 0$ leads to $\hat{U}_0 M_N \hat{U}_0^{\rm T} = 0$ where $M_N = V \hat{M}_N V^{\rm T}$
* The (1,1) element of M_N must vanish, $M_{11} = 0$, which translates into

$$M_1\,V_{11}^2 + M_2\,V_{12}^2 + M_3\,V_{13}^2 = 0$$

* Unitarity of V implies $|V_{11}|^2 + |V_{12}|^2 + |V_{13}|^2 = 1$

Some examples of U_0

* Work in the basis in which M_N is diagonal,

$$M_N = \hat{M}_N = \mathrm{diag}\left(\frac{1}{r_1}, \frac{1}{r_2}, \frac{1}{r_3}\right) m_N , \qquad r_i = \frac{m_N}{M_i}$$

* From the general form of U_0, one can derive

$$U_0 = \begin{pmatrix} a & a & i\sqrt{2}\,a \\ b & b & i\sqrt{2}\,b \\ c & c & i\sqrt{2}\,c \end{pmatrix}\mathcal{R} , \quad \begin{pmatrix} a & ia & 0 \\ b & ib & 0 \\ c & ic & 0 \end{pmatrix}\mathcal{R} , \quad \begin{pmatrix} a & 0 & ia \\ b & 0 & ib \\ c & 0 & ic \end{pmatrix}\mathcal{R} ,$$

$$\text{or} \quad \begin{pmatrix} 0 & a & ia \\ 0 & b & ib \\ 0 & c & ic \end{pmatrix}\mathcal{R} , \qquad \mathcal{R} = \mathrm{diag}\left(\sqrt{r_1}, \sqrt{r_2}, \sqrt{r_3}\right)$$

* It may sometimes be necessary to use trial and error in order to determine the right choice of U_0, with a, b, and c subject to experimental constraints.

Solutions for $U_{\nu N}$

* It is possible to find $U_{\nu N}$ satisfying $U_{\rm PMNS}\, \hat{m}_\nu\, U_{\rm PMNS}^{\rm T} = -U_{\nu N}\, \hat{M}_N\, U_{\nu N}^{\rm T}$ by adding a perturbation matrix U_δ given by

$$U_\delta = \begin{pmatrix} \delta_{11} & \delta_{12} & \delta_{13} \\ \delta_{21} & \delta_{22} & \delta_{23} \\ \delta_{31} & \delta_{32} & \delta_{33} \end{pmatrix} \mathcal{R}$$

* Since $U_0 \hat{M}_N U_0^{\rm T} = 0$,

$$m_\nu = -U_{\nu N}\, \hat{M}_N\, U_{\nu N}^{\rm T} = -U_0 \hat{M}_N U_\delta^{\rm T} - U_\delta \hat{M}_N U_0^{\rm T} - U_\delta \hat{M}_N U_\delta^{\rm T}$$

* One can neglect the last term, $U_\delta \hat{M}_N U_\delta^{\rm T}$, compared to the first two terms $\big[$except when the combination of the first two terms happens to vanish, the U_δ elements now of order $(m_\nu/m_N)^{1/2}\big]$.

* The possibility of large light-heavy mixing and tiny light-neutrino mass is natural in the presence of a conserved symmetry (lepton number L), which is softly or spontaneously broken at the TeV scale. He & Ma

Solutions with one of the light-neutrino masses being zero

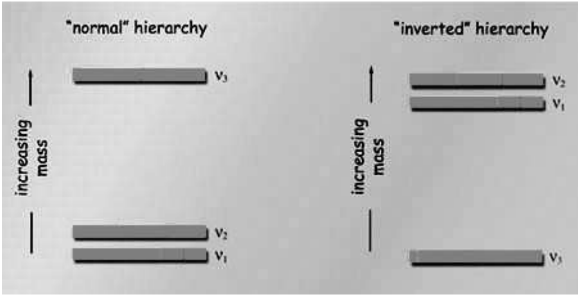
* Defining $\bar{U}_{\nu N} = U^{\dagger}_{\rm PMNS} U_{\nu N}$, $\bar{U}_0 = U^{\dagger}_{\rm PMNS} U_0$, $\bar{U}_\delta = U^{\dagger}_{\rm PMNS} U_\delta$
we have
$$\bar{U}_{\nu N} M_N \bar{U}^{\rm T}_{\nu N} = -\hat{m}_\nu \,, \qquad \hat{m}_\nu = {\rm diag}\big(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}\big)$$

* Taking, for example,
$$\bar{U}_0 = \begin{pmatrix} \bar{a} & \bar{a} & i\sqrt{2}\,\bar{a} \\ \bar{b} & \bar{b} & i\sqrt{2}\,\bar{b} \\ \bar{c} & \bar{c} & i\sqrt{2}\,\bar{c} \end{pmatrix} \mathcal{R} = U^{\dagger}_{\rm PMNS} U^a_0 \,, \qquad \bar{U}_\delta = \begin{pmatrix} \bar{\delta}_{11} & \bar{\delta}_{12} & \bar{\delta}_{13} \\ \bar{\delta}_{21} & \bar{\delta}_{22} & \bar{\delta}_{23} \\ \bar{\delta}_{31} & \bar{\delta}_{32} & \bar{\delta}_{33} \end{pmatrix} \mathcal{R}$$
we find two possible solutions with one of the light-neutrino masses required to vanish:
$$\text{(i)} \;\; \bar{a} = 0, \qquad \hat{m}_\nu = {\rm diag}\left(0,\, -1,\, \frac{\bar{c}^2}{\bar{b}^2}\right) 2\bar{b} \left(\bar{\delta}_{21} + \bar{\delta}_{22} + i\sqrt{2}\,\bar{\delta}_{23}\right) m_N \,,$$
$$\text{(ii)} \;\; \bar{c} = 0, \qquad \hat{m}_\nu = {\rm diag}\left(\frac{\bar{a}^2}{\bar{b}^2},\, -1,\, 0\right) 2\bar{b} \left(\bar{\delta}_{21} + \bar{\delta}_{22} + i\sqrt{2}\,\bar{\delta}_{23}\right) m_N \,,$$
corresponding to
(i) normal hierarchy $(m_{\nu_1} < m_{\nu_2} \ll m_{\nu_3})$ and
(ii) inverted hierarchy $(m_{\nu_3} \ll m_{\nu_1} < m_{\nu_2})$

Numerical input

* Recent fit to global neutrino data yields — Schwetz et al.

$$\Delta m_{21}^2 = m_{\nu_2}^2 - m_{\nu_1}^2 = \left(7.65_{-0.20}^{+0.23}\right) \times 10^{-5}\,\mathrm{eV}^2$$
$$\left|\Delta m_{31}^2\right| = \left|m_{\nu_3}^2 - m_{\nu_1}^2\right| = \left(2.40_{-0.11}^{+0.12}\right) \times 10^{-3}\,\mathrm{eV}^2$$



LBL

* Use $U_{\rm PMNS}$ in the tri-bimaximal form — Harrison et al.

$$U_{\rm PMNS} = U_{\rm tribi} = \begin{pmatrix} \frac{-2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} \end{pmatrix}$$

Solutions with one of the light-neutrino masses being zero

* We find

$$\text{(i)}\;\; a = 0.58\,\bar{b}\;, \qquad b \;=\; (0.58 + 1.7\,i)\bar{b}\;, \qquad c \;=\; (0.58 - 1.7\,i)\bar{b}\;,$$

$$\delta_{11} = \frac{-2.5}{10^{12}\,\bar{b}\,m_N/\mathrm{GeV}}\;, \quad \delta_{21} = \frac{(-2.5 + 7.3\,i)}{10^{12}\,\bar{b}\,m_N/\mathrm{GeV}}\;, \quad \delta_{31} = \frac{-2.5 - 7.3\,i}{10^{12}\,\bar{b}\,m_N/\mathrm{GeV}}$$

in the normal hierarchy case $(m_{\nu_1} = 0,\;\; m_{\nu_2} = 0.00875\,\mathrm{eV},\;\; m_{\nu_3} = 0.049\,\mathrm{eV})$ and

$$\text{(ii)}\;\; a = (0.58 - 0.81\,i)\bar{b}\;, \qquad b = (0.58 + 0.41)\bar{b}\;, \qquad c = (0.58 + 0.41)\bar{b}\;,$$

$$\delta_{11} = \frac{-1.4 - 2.0\,i}{10^{11}\,\bar{b}\,m_N/\mathrm{GeV}}\;, \quad \delta_{21} = \frac{-1.4 + 1.0\,i}{10^{11}\,\bar{b}\,m_N/\mathrm{GeV}}\;, \quad \delta_{31} = \frac{-1.4 + 1.0\,i}{10^{11}\,\bar{b}\,m_N/\mathrm{GeV}}$$

in the inverted hierarchy case $(m_{\nu_1} = 0.049\,\mathrm{eV},\;\; m_{\nu_2} = 0.0498\,\mathrm{eV},\;\; m_{\nu_3} = 0)$, the other δ's having been chosen to vanish.

* These examples show that large mixing (of order 0.01) of light and heavy neutrinos can occur and at the same time small neutrino masses are maintained.

Solutions with none of the light-neutrino masses being zero

* Adding another correction matrix, $U_{\alpha\beta\gamma}$, with elements of order $\left[(a,b,c)\delta_{ij}\right]^{1/2}$, one can find solutions $U_{\nu N} = U_0 + U_{\alpha\beta\gamma} + U_\delta$ for both normal and inverted hierarchies, with all 3 light-neutrino masses nonzero.
* Taking

$$\bar{U}_0 = \begin{pmatrix} 0 & \bar{a} & i\bar{a} \\ 0 & \bar{b} & i\bar{b} \\ 0 & \bar{c} & i\bar{c} \end{pmatrix} \mathcal{R} = U^\dagger_{\rm PMNS} U^d_0 \,, \quad \bar{U}_{\alpha\beta\gamma} = \begin{pmatrix} \bar{\alpha} & 0 & 0 \\ \bar{\beta} & 0 & 0 \\ \bar{\gamma} & 0 & 0 \end{pmatrix} \mathcal{R} = U^\dagger_{\rm PMNS} U_{\alpha\beta\gamma}$$

we obtain as possible solutions

$$a = -0.82\,\bar{a} \,, \qquad b = (0.41 + 0.75\,i)\bar{a} \,, \qquad c = (0.41 - 0.75\,i)\bar{a} \,,$$

$$\alpha = \frac{8.1 - 5.8\,i}{10^6\,\sqrt{m_N/{\rm GeV}}} \,, \quad \beta = \frac{-4.1 - 13\,i}{10^6\,\sqrt{m_N/{\rm GeV}}} \,, \quad \gamma = \frac{-4.1 + 1.7\,i}{10^6\,\sqrt{m_N/{\rm GeV}}} \,,$$

$$\delta_{12} = \frac{8.1 - 5.8\,i}{10^{11}\,\bar{a}\,m_N/{\rm GeV}} \,, \quad \delta_{22} = \frac{-4.1 - 5.8\,i}{10^{11}\,\bar{a}\,m_N/{\rm GeV}} \,, \quad \delta_{32} = \frac{-4.1 - 5.8\,i}{10^{11}\,\bar{a}\,m_N/{\rm GeV}}$$

in the normal-hierarchy case $\left(m_{\nu_1} = 0.0996\,\text{eV},\ m_{\nu_2} = 0.1\,\text{eV},\ m_{\nu_3} = 0.111\,\text{eV}\right)$ and the corresponding numbers in the inverted-hierarchy case, the other δ's vanishing.

Some implications for probing type-I seesaw at the LHC

* The elements of $U_{\nu N}$ must satisfy 2 other classes of measurements.
* Electroweak precision data yield, for type-I seesaw, the bounds
  $\sum_i |(U_{\nu N})_{1i}|^2 \le 0.0030, \quad \sum_i |(U_{\nu N})_{2i}|^2 \le 0.0032,$
  $\sum_i |(U_{\nu N})_{3i}|^2 \le 0.0062$ del Aguila et al.
* Data on FCNC transitions in the charged-lepton sector, such as $\mu \to e\gamma$, provide, for type-I seesaw at one-loop level, the bounds
  $\left|\sum_i (U_{\nu N})_{1i} (U_{\nu N})^*_{2i}\right| \le 0.0001, \quad \left|\sum_i (U_{\nu N})_{1i} (U_{\nu N})^*_{3i}\right| \le 0.01,$
  $\left|\sum_i (U_{\nu N})_{2i} (U_{\nu N})^*_{3i}\right| \le 0.01$ del Aguila et al., Antusch et al.
* These bounds imply the elements of $U_{\nu N}$ can be as large as 0.01.
* There are other types of solutions (obtained later for type-III seesaw) that can better evade these constraints, resulting in $U_{\nu N}$ elements of size up to 0.04.

Some implications for probing type-I seesaw at the LHC

* For $U_{\nu N}$ elements of $\mathcal{O}(0.01)$, we consider the cross section σ for $pp \to lNX$ arising from $q\bar{q}' \to W^* \to lN$ at pp center-of-mass energy of $\sqrt{s} = 14\,\text{TeV}$.
* With $U_{\nu N}$ elements ~ 0.01 (0.04), heavy neutrinos with masses up to $m_N = 115$ GeV (250 GeV) can be produced at σ of at least 1 fb.
* With 100 fb^{-1} of integrated luminosity, the production of over 3000 heavy neutrinos having $m_N = 100\,\text{GeV}$ is possible, the number of events dropping to a few for $m_N = 600\,\text{GeV}$.
* A recent analysis including background estimates suggests that N-mass values up to 150 GeV can be probed at the LHC with 30 fb^{-1} luminosity.

del Aguila et al.

Type-III seesaw

Foot et al.

* The seesaw mechanism is realized by introducing weak-$\mathrm{SU}(2)_\mathrm{L}$ triplets of right-handed heavy leptons having zero hypercharge.
* The components of each triplet Σ and its charge conjugate Σ^c

$$\Sigma = \begin{pmatrix} N^0/\sqrt{2} & E^+ \\ E^- & -N^0/\sqrt{2} \end{pmatrix}, \qquad \Sigma^c = C\bar{\Sigma}^\mathrm{T} = \begin{pmatrix} N^{0c}/\sqrt{2} & E^{-c} \\ E^{+c} & -N^{0c}/\sqrt{2} \end{pmatrix}$$

* The Lagrangian for Σ

$$\begin{aligned} \mathcal{L}_\mathrm{III} &= \bar{E} i \partial\!\!\!/ E + \overline{N_R^0} i \partial\!\!\!/ N_R^0 - \bar{E} M_\Sigma E - \tfrac{1}{2}\left[\overline{N_R^0} M_\Sigma \left(N_R^0\right)^c + \mathrm{H.c.}\right] \\ &+ g\left[\overline{N_R^0}\, W\!\!\!/^{+} E_R + \overline{\left(N_R^0\right)^c}\, W\!\!\!/^{+} E_L + \mathrm{H.c.}\right] - g\,\bar{E}\, W\!\!\!/_3 E \\ &- \left[\tfrac{1}{\sqrt{2}}(v+h)\overline{N_R^0} Y_\Sigma \nu_L + (v+h)\bar{E} Y_\Sigma l_L + \mathrm{H.c.}\right] \end{aligned}$$

$E = \left(E_R^+\right)^c + E_R^-$, $W_3^\mu = -s_\mathrm{w} A^\mu + c_\mathrm{w} Z^\mu$, $s_\mathrm{w} = \sin\theta_\mathrm{W}$, $N_R = N$,
$E_{L,R} = P_{L,R} E$, $P_{L,R} = \tfrac{1}{2}(1 \mp \gamma_5)$

Masses of neutrinos in type-III seesaw

* The neutrino-mass matrix has the seesaw form given by

$$\mathcal{L}'_{\rm mass} = -\tfrac{1}{2}\left(\overline{(\nu_L)^c} \quad \overline{N^0}\right)\begin{pmatrix} 0 & Y_\Sigma^{\rm T} v/\sqrt{2} \\ Y_\Sigma v/\sqrt{2} & M_\Sigma \end{pmatrix}\begin{pmatrix} \nu_L \\ N^{0c} \end{pmatrix} + \text{ H.c.}$$

* The heavy charged leptons mix with the SM charged leptons resulting in the mass-matrix term

$$\mathcal{L}''_{\rm mass} = -\left(\overline{l_R} \quad \overline{E_R}\right)\begin{pmatrix} m_l & 0 \\ Y_\Sigma v & M_\Sigma \end{pmatrix}\begin{pmatrix} l_L \\ E_L \end{pmatrix} + \text{ H.c.}$$

* One can diagonalize the (6×6) mass matrices by transforming from the weak eigenstates to mass eigenstates using the relations

$$\begin{pmatrix} \nu_L \\ N^{0c} \end{pmatrix} = U \begin{pmatrix} \nu_{mL} \\ N_{mL} \end{pmatrix}, \qquad \begin{pmatrix} l_{L,R} \\ E_{L,R} \end{pmatrix} = U_{L,R} \begin{pmatrix} l_{mL,mR} \\ E_{mL,mR} \end{pmatrix}$$

with

$$U = \begin{pmatrix} U_{\nu\nu} & U_{\nu N} \\ U_{N\nu} & U_{NN} \end{pmatrix}, \quad U_L = \begin{pmatrix} U_{Lll} & U_{LlE} \\ U_{LEl} & U_{LEE} \end{pmatrix}, \quad U_R = \begin{pmatrix} U_{Rll} & U_{RlE} \\ U_{REl} & U_{REE} \end{pmatrix}$$

Interactions of N & E in type-III seesaw

* From the diagonalization, to 2nd order in $Y_\Sigma v M_\Sigma^{-1}$ and/or $m_l M_\Sigma^{-1}$

$$U_{\nu\nu} = \left(1 - \tfrac{1}{2}\,\epsilon\right) U_{\rm PMNS}\ , \quad U_{\nu N} = \tfrac{1}{\sqrt{2}}\, Y_\Sigma^\dagger M_\Sigma^{-1} v\ ,$$

$$U_{N\nu} = \tfrac{-1}{\sqrt{2}}\, M_\Sigma^{-1} Y_\Sigma U_{\nu\nu}\, v\ , \quad U_{NN} = 1 - \tfrac{1}{2}\,\epsilon'\ ,$$

$$U_{Lll} = 1 - \epsilon\ , \qquad U_{LlE} = Y_\Sigma^\dagger M_\Sigma^{-1} v\ , \quad U_{LEl} = -M_\Sigma^{-1} Y_\Sigma v\ , \quad U_{LEE} = 1 - \epsilon'\ ,$$

$$U_{Rll} = 1\ , \quad U_{RlE} = m_l Y_\Sigma^\dagger M_\Sigma^{-2} v\ , \quad U_{REl} = -M_\Sigma^{-2} Y_\Sigma m_l v\ , \quad U_{REE} = 1\ ,$$

$$\epsilon \ \equiv\ \tfrac{1}{2}\, Y_\Sigma^\dagger M_\Sigma^{-2} Y_\Sigma v^2 \ =\ U_{\nu N} U_{\nu N}^\dagger\ , \qquad \epsilon' \ \equiv\ \tfrac{1}{2}\, M_\Sigma^{-1} Y_\Sigma Y_\Sigma^\dagger M_\Sigma^{-1} v^2 \ =\ U_{\nu N}^\dagger U_{\nu N}$$

* From $\mathcal{L}_{\rm III}$, the interactions of E, at leading order, are described by

$$\mathcal{L}_E \ = \ -g\, \overline{\left(\nu_{mL}\right)^c}\, \not{W}^+ U_{\rm PMNS}^{\rm T} U_{\nu N}^* E_{mR} + \frac{g}{\sqrt{2}\, c_{\rm w}}\, \overline{l_{mL}}\, \not{Z} U_{\nu N} E_{mL}$$
$$- \ \frac{g}{\sqrt{2}\, m_W}\, \overline{l_{mL}}\, U_{\nu N} M_\Sigma E_{mR} h \ + \ {\rm H.c.}\ .$$

The interactions of N are the same as those in type-I seesaw.

Charged-lepton FCNC's in type-III seesaw

* In terms of the mass eigenstates, $\mathcal{L}_{\rm III}$ also contains the interactions of the light charged leptons with the Z and Higgs bosons

$$\mathcal{L}_l = \frac{g}{c_{\rm w}}\,\bar{l}_m\gamma^\mu\left[\left(-\tfrac{1}{2}+s_{\rm w}^2-\epsilon\right)P_L+s_{\rm w}^2\,P_R\right]l_m\,Z_\mu + \frac{g}{2m_W}\,\bar{l}_m\left[m_l(3\epsilon-1)P_L+(3\epsilon-1)m_l\,P_R\right]l_m\,h\ .$$

* The off-diagonal elements of ϵ are new sources of tree-level FCNC's in the charged-lepton sector.

* Existing data on FCNC transitions involving ordinary leptons yield strict bounds on the off-diagonal elements of ϵ.
  - $|\epsilon_{12}| < 1.7\times10^{-7}$ from μ-e conversion in atomic nuclei — Abada et al.
  - $|\epsilon_{13}| < 4.2\times10^{-4}$ from $\tau\to\pi^0 e$
  - $|\epsilon_{23}| < 4.9\times10^{-4}$ from $\tau\to\mu\bar{\mu}\mu$ — He & Oh

FCNC constraints

* Since $\epsilon = U_{\nu N} U_{\nu N}^\dagger$, can we find $U_{\nu N}$ with large enough elements without resulting in FCNC τ decays being too suppressed?

* The U_0 results in type-I seesaw give, as examples,

$$\text{(i)} \;\; \epsilon = \begin{pmatrix} 0.67 & -0.33+0.61\,i & -0.33-0.61\,i \\ -0.33-0.61\,i & 0.72 & -0.39+0.61\,i \\ -0.33+0.61\,i & -0.39-0.61\,i & 0.72 \end{pmatrix} |\bar{a}|^2 \left(r_2+r_3\right) \,,$$

$$\text{(ii)} \;\; \epsilon = \begin{pmatrix} 0.67 & -0.33+0.54\,i & -0.33-0.54\,i \\ -0.33-0.54\,i & 0.60 & -0.27+0.54\,i \\ -0.33+0.54\,i & -0.27-0.54\,i & 0.60 \end{pmatrix} |\bar{a}|^2 \left(r_2+r_3\right)$$

* Since in these cases ϵ_{12}, ϵ_{13}, and ϵ_{23} have the same order of magnitude, the strictest constraint, $|\epsilon_{12}| < 1.7 \times 10^{-7}$, would translate into ϵ_{13} and ϵ_{23} values less than 10^{-6}, which would make FCNC's in τ decays too small to be interesting.

Solutions with suppressed ϵ_{12} and large ϵ_{23}

* We obtain a desired solution with the choice $U_{\nu N} = U_0^e + U_{\alpha\beta\gamma}^e + U_\delta^e$,

$$U_0^e = \begin{pmatrix} 0 & 0 & 0 \\ 0 & a & ia \\ 0 & b & ib \end{pmatrix} \mathcal{R} \, , \quad U_{\alpha\beta\gamma}^e = \begin{pmatrix} \alpha & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \mathcal{R} \, , \quad U_\delta^e = \begin{pmatrix} 0 & \delta_{12} & 0 \\ 0 & \delta_{22} & 0 \\ 0 & \delta_{32} & 0 \end{pmatrix} \mathcal{R}$$

* The results are $b = a \, , \quad \delta_{22} = \delta_{32} = \dfrac{a\,\delta_{12} + \alpha^2}{4a}$

  corresponding to an inverted-hierarchy case with $m_{\nu_3} = 0$.

* Numerically, $\alpha^2 = -4.9 \times 10^{-11} \, \dfrac{\text{GeV}}{m_N} \, , \quad \delta_{12} = \dfrac{-2.6 \times 10^{-13}\,\text{GeV}}{a\,m_N}$

  implying $$\epsilon = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} |a|^2 \, (r_2 + r_3)$$

* The constraint $|\epsilon_{23}| = |\epsilon_{\mu\tau}| < 4.9 \times 10^{-4}$ from $\tau \to \mu\bar{\mu}\mu$ decays translates into $|a|\sqrt{r_2 + r_3} < 2.2 \times 10^{-2}$

Solutions with suppressed ϵ_{12} and large ϵ_{13}

* We obtain a desired solution with the choice $U_{\nu N} = U_0^f + U_{\alpha\beta\gamma}^f + U_\delta^e$,

$$U_0^f = \begin{pmatrix} 0 & a & ia \\ 0 & 0 & 0 \\ 0 & b & ib \end{pmatrix} \mathcal{R} , \qquad U_{\alpha\beta\gamma}^f = \begin{pmatrix} \alpha & 0 & 0 \\ \beta & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \mathcal{R}$$

* The results are

$$\epsilon = \begin{pmatrix} 1 & 0 & 0.001 - 1.0\,i \\ 0 & 0 & 0 \\ 0.001 + 1.0\,i & 0 & 1.1 \end{pmatrix} |a|^2 \left(r_2 + r_3\right) ,$$

in the normal-hierarchy case $\left(m_{\nu_1} = 0.0996\,\text{eV},\ m_{\nu_2} = 0.1\,\text{eV},\ m_{\nu_3} = 0.111\,\text{eV}\right)$ and

$$\epsilon = \begin{pmatrix} 1 & 0 & 0.001 + 0.96\,i \\ 0 & 0 & 0 \\ 0.001 - 0.96\,i & 0 & 0.93 \end{pmatrix} |a|^2 \left(r_2 + r_3\right)$$

in the inverted-hierarchy case $\left(m_{\nu_1} = 0.0996\,\text{eV},\ m_{\nu_2} = 0.1\,\text{eV},\ m_{\nu_3} = 0.0867\,\text{eV}\right)$

* The bound $|\epsilon_{13}| = |\epsilon_{e\tau}| < 4.2 \times 10^{-4}$ from $\tau \to \pi^0 e$ then implies $|a|\sqrt{r_2 + r_3} < 2.0 \times 10^{-2}$ in the two cases.

Some implications for testing type-III seesaw at the LHC

* For type-III seesaw, the electroweak precision data yield $\sum_i |(U_{\nu N})_{1i}|^2 \le 0.00036$, $\sum_i |(U_{\nu N})_{2i}|^2 \le 0.00029$, del Aguila et al. $\sum_i |(U_{\nu N})_{3i}|^2 \le 0.00073$, which translate into $|a|\sqrt{r_2 + r_3} < 1.9 \times 10^{-2}$, comparable to the numbers from FCNC constraints.

* With the nonzero $U_{\nu N}$ elements ~ 0.01, the cross-section of $pp \to lNX$ due to $q\bar{q}' \to W^* \to lN$ exceeds 1 fb for N masses up to $m_N = 115\,\text{GeV}$.
* $100\,\text{fb}^{-1}$ of luminosity can yield about 200 N's with $m_N = 100\,\text{GeV}$ and at least a few of them with $m_N = 300\,\text{GeV}$.

* The heavy charged lepton E can be produced via $q\bar{q} \to (Z^*, h^*) \to l^\pm E^\mp$ and $q\bar{q}' \to W^* \to \nu E$
* The $pp \to lEX$ cross-section stays $> 1\,\text{fb}$ for masses up to $m_E = 115\,\text{GeV}$.
* With $100\,\text{fb}^{-1}$ of luminosity, the production of more than 200 E's with $m_E = 100\,\text{GeV}$ is possible and a few of them with $m_N = 300\,\text{GeV}$.

* These single-production channels can provide information complementary to $q\bar{q} \to Z^* \to E^+E^-$ and $q\bar{q}' \to W^* \to NE$.

Conclusions

* With more than 1 generation of light and heavy neutrinos, it is possible in certain special circumstances to have light-heavy mixing sufficiently large to allow seesaw models to be directly tested at colliders.
* We have explored this possibility further and considered specific examples in detail in the context of type-I seesaw.
* We have studied its implications for the single production of the heavy neutrinos at the LHC via the main channel $q\bar{q}' \to W^* \to lN$.
* We then extend the discussion to type-III seesaw, which has richer phenomenology due to presence of the charged partners of the heavy neutrinos, and examine the implications for their production at the LHC.
* In type-III seesaw, the solutions we find also make it possible to have sizable FCNC effects in processes involving ordinary charged leptons.
* All our estimates above suggest that there are interesting prospects for discovering these heavy particles at the LHC and should give further motivation for carrying out dedicated experimental searches of them.