

“Neutrino Masses in the Supersymmetric
 $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ Model
with right-handed neutrinos”

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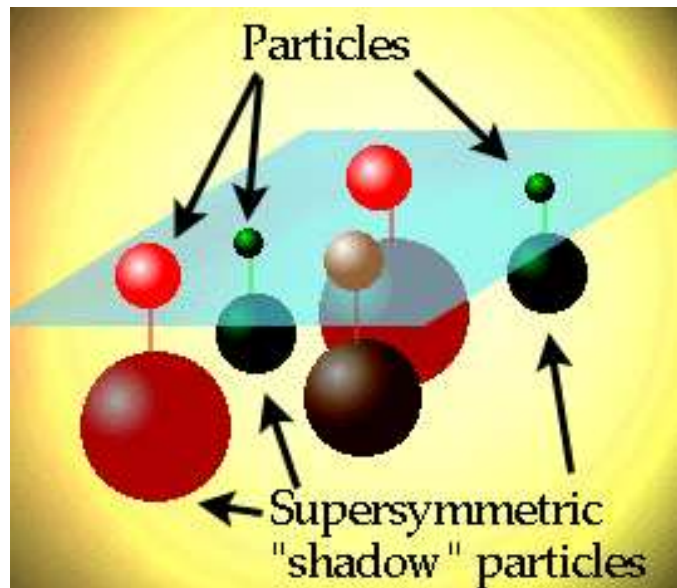
Motivation to study 331 Models

1. At low energies they coincide with the SM;
2. These sort of model are anomaly free only if the family number is 3 or any multiple of 3;
3. They have bileptons charged vector bosons;
4. They have new heavy quarks;
5. Since one generation of quarks is treated differently from the others this may lead to a natural explanation for the large mass of the top quark;

Motivation to study SUSY

- Unify bosons and fermions

$$\begin{aligned} Q|\text{boson}\rangle &= |\text{fermion}\rangle \\ Q|\text{fermion}\rangle &= |\text{boson}\rangle \end{aligned}$$

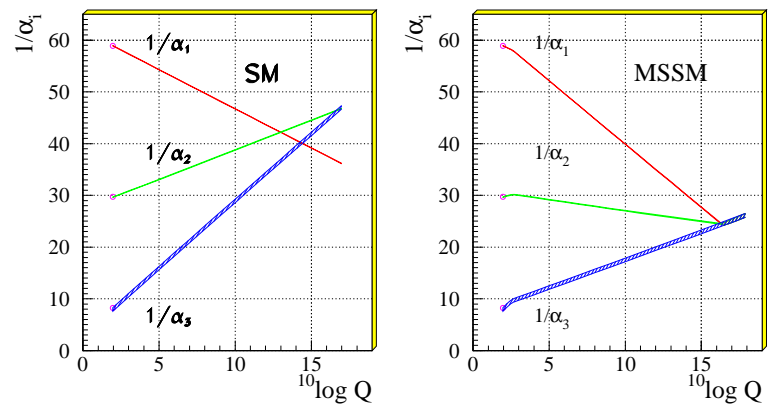


- SUSY local unify with Gravity (Supergravity)

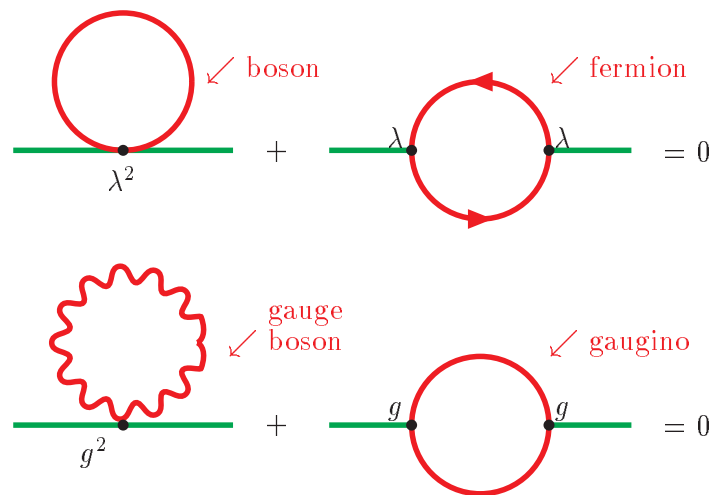
$$\text{spin}2 \rightarrow \text{spin}\frac{3}{2} \rightarrow \text{spin}1 \rightarrow \text{spin}\frac{1}{2} \rightarrow \text{spin}0$$

- Unify all the gauge constants

Unification of the Coupling Constants
in the SM and the minimal MSSM



- Solve the hierarchy problem



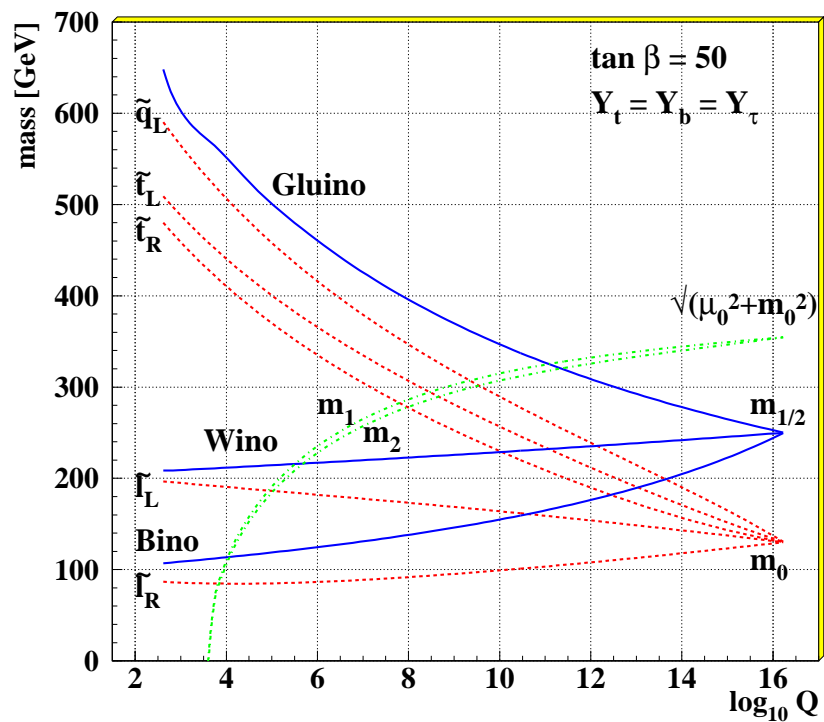
cancel same if susy is broken

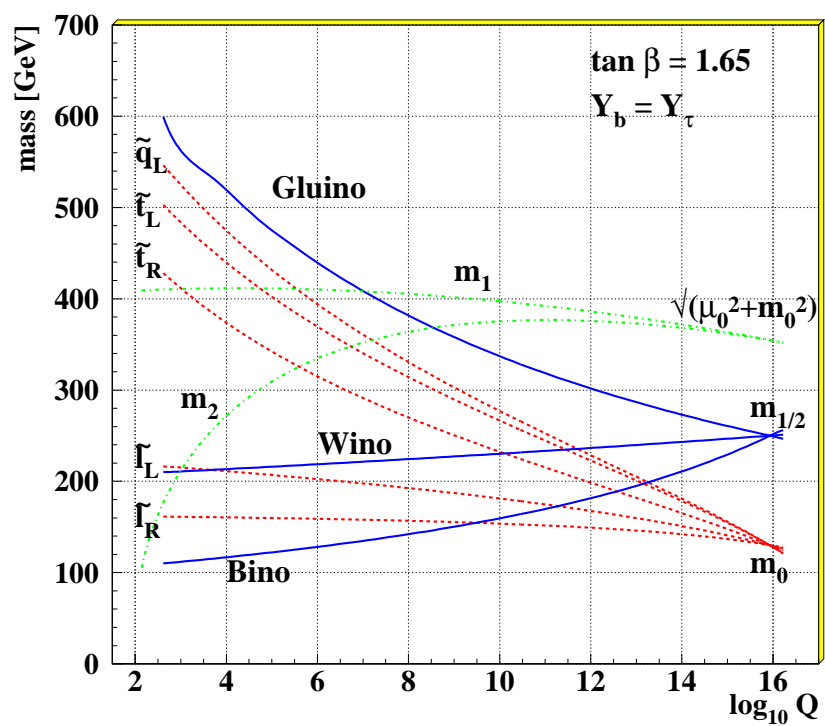
$$\sum_{bosons} m^2 - \sum_{fermions} m^2 = M_{SUSY}^2 \leq 1\text{TeV} \quad (1)$$

Higgs Boson

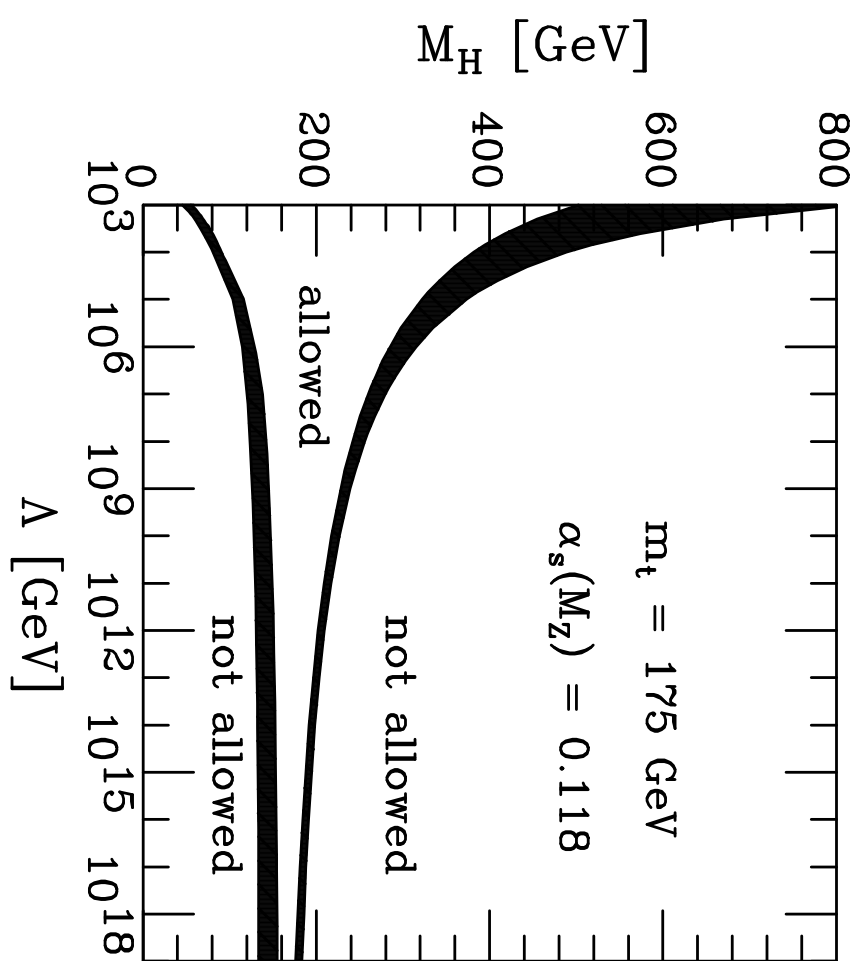
$$\delta M_h^2 \sim g^2 M_{SUSY}^2 \sim M_h^2 \quad (2)$$

Understood Spontaneous Symmetry Breaking mechanism $\mu > 0 \rightarrow \mu < 0$





$m_t \sim 175$ GeV imply $M_h \sim 200$ GeV



The Model

Lepton sector

- Triplets

$$L_{aL} = \begin{pmatrix} \nu_a \\ l_a \\ \nu_a^c \end{pmatrix}_L \sim (\mathbf{1}, \mathbf{3}, -1/3), \quad (3)$$

$$a = e, \mu, \tau$$

- Singlets

$$l_{aL}^c \sim (\mathbf{1}, \mathbf{1}, \mathbf{1}). \quad (4)$$

Quark sector

- Antitriplets

$$Q_{\alpha L} = \begin{pmatrix} d_{\alpha} \\ u_{\alpha} \\ D_{\alpha} \end{pmatrix}_L \sim (\mathbf{3}, \mathbf{3}^*, 0), \quad \alpha = 1, 2; \quad (5)$$

- Triplets

$$Q_{3L} = \begin{pmatrix} u_3 \\ d_3 \\ T \end{pmatrix}_L \sim (\mathbf{3}, \mathbf{3}, 1/3), \quad (6)$$

- Singlets

$$u_{\alpha L}^c \sim (\mathbf{3}^*, \mathbf{1}, -2/3), \quad d_{\alpha L}^c, D_{\alpha L}^c \sim (\mathbf{3}^*, \mathbf{1}, 1/3). \quad (7)$$

$$u_{3L}^c, T_L^c \sim (\mathbf{3}^*, \mathbf{1}, -2/3), \quad d_{3L}^c \sim (\mathbf{3}^*, \mathbf{1}, 1/3). \quad (8)$$

- triplets

$$\begin{aligned} \eta &= \begin{pmatrix} \eta_1^0 \\ \eta_1^- \\ \eta_2^0 \end{pmatrix}, \quad \chi = \begin{pmatrix} \chi_1^0 \\ \chi_1^- \\ \chi_2^0 \end{pmatrix} \sim (\mathbf{1}, \mathbf{3}, -1/3), \\ \rho &= \begin{pmatrix} \rho_1^+ \\ \rho_1^0 \\ \rho_2^+ \end{pmatrix} \sim (\mathbf{1}, \mathbf{3}, 2/3), \end{aligned} \quad (9)$$

- Antitriplets

$$\begin{aligned} \eta' &= \begin{pmatrix} \eta_1'^0 \\ \eta_1'^+ \\ \eta_2'^0 \end{pmatrix}, \quad \chi' = \begin{pmatrix} \chi_1'^0 \\ \chi_1'^+ \\ \chi_2'^0 \end{pmatrix} \sim \left(\mathbf{1}, \mathbf{3}^*, \frac{1}{3}\right); \\ \rho' &= \begin{pmatrix} \rho_1'^- \\ \rho_1'^0 \\ \rho_2'^- \end{pmatrix} \sim \left(\mathbf{1}, \mathbf{3}^*, -\frac{2}{3}\right). \end{aligned} \quad (10)$$

vev

$$\begin{aligned} \langle \eta \rangle &= \begin{pmatrix} v \\ 0 \\ 0 \end{pmatrix}, & \langle \chi \rangle &= \begin{pmatrix} 0 \\ 0 \\ w \end{pmatrix}, \\ \langle \rho \rangle &= \begin{pmatrix} 0 \\ u \\ 0 \end{pmatrix}, \\ \langle \eta' \rangle &= \begin{pmatrix} v' \\ 0 \\ 0 \end{pmatrix}, & \langle \chi' \rangle &= \begin{pmatrix} 0 \\ 0 \\ w' \end{pmatrix}, \\ \langle \rho' \rangle &= \begin{pmatrix} 0 \\ u' \\ 0 \end{pmatrix}. \end{aligned} \tag{11}$$

Squarks, Sleptons

$$\tilde{Q}_{\alpha L} = \begin{pmatrix} \tilde{d}_{\alpha} \\ \tilde{u}_{\alpha} \\ \tilde{D}_{\alpha} \end{pmatrix}_L \sim (\mathbf{3}, \mathbf{3}^*, 0), \quad \alpha = 1, 2$$

$$\tilde{Q}_{3L} = \begin{pmatrix} \tilde{u}_3 \\ \tilde{d}_3 \\ \tilde{T} \end{pmatrix}_L \sim (\mathbf{3}, \mathbf{3}, 1/3),$$

$$\tilde{L}_{aL} = \begin{pmatrix} \tilde{\nu}_a \\ \tilde{l}_a \\ \tilde{\nu}_a^c \end{pmatrix}_L \sim (\mathbf{1}, \mathbf{3}, -1/3), \quad a = e, \mu, \tau$$

$$\tilde{l}_{aL}^c \sim (\mathbf{1}, \mathbf{1}, 1),$$

$$\tilde{u}_{iL}^c, \tilde{T}_L^c \sim (\mathbf{3}^*, \mathbf{1}, -2/3), \quad i = 1, 2, 3$$

$$\tilde{d}_{iL}^c, \tilde{D}_{\alpha L}^c \sim (\mathbf{3}^*, \mathbf{1}, 1/3),$$

higgsinos

$$\begin{aligned}
\tilde{\eta} &= \begin{pmatrix} \tilde{\eta}_1^0 \\ \tilde{\eta}^- \\ \tilde{\eta}_2^0 \end{pmatrix}, \quad \tilde{\chi} = \begin{pmatrix} \tilde{\chi}_1^0 \\ \tilde{\chi}^- \\ \tilde{\chi}_2^0 \end{pmatrix} \sim (1, \mathbf{3}, -1/3), \\
\tilde{\rho} &= \begin{pmatrix} \tilde{\rho}_1^+ \\ \tilde{\rho}^0 \\ \tilde{\rho}_2^+ \end{pmatrix} \sim (1, \mathbf{3}, 2/3), \\
\tilde{\eta}' &= \begin{pmatrix} \tilde{\eta}_1'^0 \\ \tilde{\eta}'^+ \\ \tilde{\eta}_2'^0 \end{pmatrix}, \quad \tilde{\chi}' = \begin{pmatrix} \tilde{\chi}_1'^0 \\ \tilde{\chi}'^+ \\ \tilde{\chi}_2'^0 \end{pmatrix} \sim (1, \mathbf{3}^*, 1/3), \\
\tilde{\rho}' &= \begin{pmatrix} \tilde{\rho}_1'^- \\ \tilde{\rho}'^0 \\ \tilde{\rho}_2'^- \end{pmatrix} \sim (1, \mathbf{3}^*, -2/3). \tag{12}
\end{aligned}$$

Boson Sector

Particle	Spin	Superpartner	Spin
(U(1)) V'_m	1	λ_B	$\frac{1}{2}$
(SU(3)) V_m^a	1	λ_A^a	$\frac{1}{2}$
(SU(3)) g_m^a	1	λ_C^a	$\frac{1}{2}$

Lagrangian

$$\mathcal{L}_{331S} = \mathcal{L}_{SUSY} + \mathcal{L}_{\text{soft}},$$

supersymmetric Lagrangian

$$\mathcal{L}_{SUSY} = \mathcal{L}_{\text{Lepton}} + \mathcal{L}_{\text{Quark}} + \mathcal{L}_{\text{Gauge}} + \mathcal{L}_{\text{Scalar}}$$

$$\mathcal{L}_{\text{Leptons}} = \int d^4\theta \left[\hat{L}_{\alpha L} e^{2g\hat{V} - \frac{g'}{3}\hat{V}'} \hat{L}_{\alpha L} + \hat{l}_{\alpha L}^c e^{g'\hat{V}'} \hat{l}_{\alpha L}^c \right],$$

$$\begin{aligned} \mathcal{L}_{\text{Quarks}} = & \int d^4\theta \left[\hat{Q}_{\alpha L} e^{2(g_s\hat{V}_C + g\hat{V})} \hat{Q}_{\alpha L} \right. \\ & + \hat{Q}_{3L} e^{[2(g_s\hat{V}_C + g\hat{V}) + \frac{g'}{3}\hat{V}']} \hat{Q}_{3L} \\ & + \hat{u}_{iL}^c e^{[2g_s\hat{V}_C - \frac{2g'}{3}\hat{V}']} \hat{u}_{iL}^c \\ & + \hat{d}_{iL}^c e^{[2g_s\hat{V}_C + \frac{g'}{3}\hat{V}']} \hat{d}_{iL}^c \\ & + \hat{T}_L^c e^{[2g_s\hat{V}_C - \frac{2g'}{3}\hat{V}']} \hat{T}_L^c \\ & \left. + \hat{D}_{\alpha L}^c e^{[2g_s\hat{V}_C + \frac{g'}{3}\hat{V}']} \hat{D}_{\alpha L}^c \right], \end{aligned}$$

$$\begin{aligned} \mathcal{L}_{\text{Gauge}} &= \frac{1}{4} \int d^2\theta \left[\mathcal{W}_C \mathcal{W}_C + \mathcal{W} \mathcal{W} + \mathcal{W}' \mathcal{W}' \right] \\ &+ \frac{1}{4} \int d^2\bar{\theta} \left[\bar{\mathcal{W}}_C \bar{\mathcal{W}}_C + \bar{\mathcal{W}} \bar{\mathcal{W}} + \bar{\mathcal{W}}' \bar{\mathcal{W}}' \right], \end{aligned}$$

$$\mathcal{W}_{\zeta C} = -\frac{1}{8g_s} \bar{D} \bar{D} e^{-2g_s \hat{V}_C} D_{\zeta} e^{2g_s \hat{V}_C},$$

$$\mathcal{W}_{\zeta} = -\frac{1}{8g} \bar{D} \bar{D} e^{-2g \hat{V}} D_{\zeta} e^{2g \hat{V}},$$

$$\mathcal{W}'_{\zeta} = -\frac{1}{4} \bar{D} \bar{D} D_{\zeta} \hat{V}', \quad \zeta = 1, 2.$$

$$\begin{aligned} \mathcal{L}_{\text{Escalar}} &= \int d^4\theta \left[\hat{\eta} e^{[2g \hat{V} - \frac{g'}{3} \hat{V}']} \hat{\eta} + \hat{\chi} e^{[2g \hat{V} - \frac{g'}{3} \hat{V}']} \hat{\chi} \right. \\ &+ \hat{\rho} e^{[2g \hat{V} + \frac{2g'}{3} \hat{V}']} \hat{\rho} + \hat{\eta}' e^{[2g \hat{V} + \frac{g'}{3} \hat{V}']} \hat{\eta}' \\ &+ \hat{\chi}' e^{[2g \hat{V} + \frac{g'}{3} \hat{V}']} \hat{\chi}' + \hat{\rho}' e^{[2g \hat{V} - \frac{2g'}{3} \hat{V}']} \hat{\rho}' \left. \right] \\ &+ \int d^2\theta W + \int d^2\bar{\theta} \bar{W}, \end{aligned}$$

W is the superpotential

$$W = \frac{W_2}{2} + \frac{W_3}{3}, \quad \bar{W} = \frac{\bar{W}_2}{2} + \frac{\bar{W}_3}{3},$$

$$W_2 = \mu_{0a} \hat{L}_a \hat{\eta}' + \mu_{1a} \hat{L}_a \hat{\chi}' + \mu_\eta \hat{\eta} \hat{\eta}' + \mu_\chi \hat{\chi} \hat{\chi}' + \mu_1 \hat{\eta} \hat{\chi}' \\ + \mu_2 \hat{\chi} \hat{\eta}' + \mu_\rho \hat{\rho} \hat{\rho}',$$

$$W_3 = \lambda_{1ab} \hat{L}_a \hat{\rho}' \hat{l}_b^c + \lambda_{2a} \epsilon \hat{L}_a \hat{\chi} \hat{\rho} + \lambda_{3a} \epsilon \hat{L}_a \hat{\eta} \hat{\rho} \\ + \lambda_{4ab} \epsilon \hat{L}_a \hat{L}_b \hat{\rho} + \kappa_{1i} \hat{Q}_3 \hat{\eta}' \hat{u}_i^c + \kappa'_1 \hat{Q}_3 \hat{\eta}' \hat{T}^c \\ + \kappa_{2i} \hat{Q}_3 \hat{\chi}' \hat{u}_i^c + \kappa'_2 \hat{Q}_3 \hat{\chi}' \hat{T}^c + \kappa_{3\alpha i} \hat{Q}_\alpha \hat{\eta} \hat{d}_i^c \\ + \kappa'_{3\alpha\beta} \hat{Q}_\alpha \hat{\eta} \hat{D}_\beta^c + \kappa_{4\alpha i} \hat{Q}_\alpha \hat{\rho} \hat{u}_i^c + \kappa'_{4\alpha} \hat{Q}_\alpha \hat{\rho} \hat{T}^c \\ + \kappa_{5i} \hat{Q}_3 \hat{\rho}' \hat{d}_i^c + \kappa'_{5\beta} \hat{Q}_3 \hat{\rho}' \hat{d}_\beta^c + \kappa_{6\alpha i} \hat{Q}_\alpha \hat{\chi} \hat{d}_i^c \\ + \kappa'_{6\alpha\beta} \hat{Q}_\alpha \hat{\chi} \hat{D}_\beta^c + f_1 \epsilon \hat{\rho} \hat{\chi} \hat{\eta} + f'_1 \epsilon \hat{\rho}' \hat{\chi}' \hat{\eta}' \\ + \zeta_{\alpha\beta\gamma} \epsilon \hat{Q}_\alpha \hat{Q}_\beta \hat{Q}_\gamma + \lambda'_{\alpha ai} \hat{Q}_\alpha \hat{L}_a \hat{d}_i^c + \lambda''_{ijk} \hat{d}_i^c \hat{u}_j^c \hat{d}_k^c \\ + \xi_{1ij\beta} \hat{d}_i^c \hat{u}_j^c \hat{D}_\beta^c + \xi_{2\alpha a\beta} \hat{Q}_\alpha \hat{L}_a \hat{D}_\beta^c + \xi_{3i\beta} \hat{d}_i^c \hat{T}^c \hat{D}_\beta^c \\ + \xi_{4ij} \hat{d}_i^c \hat{T}^c \hat{d}_j^c + \xi_{5\alpha i\beta} \hat{D}_\alpha^c \hat{u}_i^c \hat{D}_\beta^c + \xi_{6\alpha\beta} \hat{D}_\alpha^c \hat{T}^c \hat{D}_\beta^c$$

$$\begin{aligned}
W_{RC} = & \frac{\mu_\eta}{2} \hat{\eta} \hat{\eta}' + \frac{\mu_\chi}{2} \hat{\chi} \hat{\chi}' + \frac{\mu_\rho}{2} \hat{\rho} \hat{\rho}' + \frac{\mu_2}{2} \hat{\eta} \hat{\chi}' + \frac{\mu_3}{2} \hat{\chi} \hat{\eta}' \\
& + \frac{1}{3} \left[\lambda_{1ab} \hat{L}_a \hat{\rho}' \hat{l}_b^c + \kappa_{1i} \hat{Q}_3 \hat{\eta}' \hat{u}_i^c + \kappa'_1 \hat{Q}_3 \hat{\eta}' \hat{u}'^c \right. \\
& + \kappa_{2i} \hat{Q}_3 \hat{\chi}' \hat{u}_i^c + \kappa'_2 \hat{Q}_3 \hat{\chi}' \hat{u}'^c + \kappa_{3\alpha i} \hat{Q}_\alpha \hat{\eta} \hat{d}_i^c \\
& + \kappa'_{3\alpha\beta} \hat{Q}_\alpha \hat{\eta} \hat{d}_\beta^c + \kappa_{4\alpha i} \hat{Q}_\alpha \hat{\rho} \hat{u}_i^c + \kappa'_{4\alpha} \hat{Q}_\alpha \hat{\rho} \hat{u}'^c \\
& + \kappa_{5i} \hat{Q}_3 \hat{\rho}' \hat{d}_i^c + \kappa'_{5\beta} \hat{Q}_3 \hat{\rho}' \hat{d}_\beta^c + \kappa_{6\alpha i} \hat{Q}_\alpha \hat{\chi} \hat{d}_i^c \\
& \left. + \kappa'_{6\alpha\beta} \hat{Q}_\alpha \hat{\chi} \hat{d}_\beta^c + f_1 \hat{\epsilon} \hat{\rho} \hat{\chi} \hat{\eta} + f'_1 \hat{\epsilon}' \hat{\rho}' \hat{\chi}' \hat{\eta}' \right]
\end{aligned}$$

$$R\text{-parity} = (-1)^{2S} (-1)^{3(B+\mathcal{L})}, \quad (13)$$

quarks and charged leptons get their masses
neutrinos remain massless

<i>Triplet</i>	L	Q_3	χ	η	ρ
\mathcal{B} charge	0	$\frac{1}{3}$	0	0	0
\mathcal{L} charge	$\frac{1}{3}$	$-\frac{2}{3}$	$\frac{4}{3}$	$-\frac{2}{3}$	$-\frac{2}{3}$

(14)

<i>Anti – Triplet</i>	Q_α	χ'	η'	ρ'
\mathcal{B} charge	$\frac{1}{3}$	0	0	0
\mathcal{L} charge	$\frac{2}{3}$	$-\frac{4}{3}$	$\frac{2}{3}$	$\frac{2}{3}$

(15)

<i>Singlet</i>	l^c	u^c	d^c	u'^c	d'^c
\mathcal{B} charge	0	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$
\mathcal{L} charge	-1	0	0	2	-2

(16)

Discrete Symmetry for Proton Stability and Neutrino Masses in SUSY331RN

$$\begin{aligned}
 n_L &= n_\eta = n_\chi = \frac{1}{2}, \\
 n_{\eta'} &= n_{\chi'} = n_u = n_{u'} = -\frac{1}{2}, \\
 n_{Q_3} &= n_{\rho'} = 1, \quad n_d = n_{d'} = -2, \\
 n_\rho &= -1, \quad n_{Q_\alpha} = \frac{3}{2}, \quad n_l = -\frac{3}{2}, \quad (17)
 \end{aligned}$$

$$\begin{aligned}
 W &= W_{RC} + \frac{1}{2} \left(\mu_{0a} \hat{L}_a \hat{\eta}' + \mu_{1a} \hat{L}_a \hat{\chi}' \right) + \frac{1}{3} \left(\lambda_{1ab} \hat{L}_a \hat{\rho}' \hat{l}_b^c \right. \\
 &+ \lambda_{2a} \epsilon \hat{L}_a \hat{\chi} \hat{\rho} + \lambda_{3a} \epsilon \hat{L}_a \hat{\eta} \hat{\rho} + \lambda_{4ab} \epsilon \hat{L}_a \hat{L}_b \hat{\rho} \\
 &\left. + \lambda'_{\alpha ai} \hat{Q}_\alpha \hat{L}_a \hat{d}_i^c + \xi_{2\alpha a\beta} \hat{Q}_\alpha \hat{L}_a \hat{d}_\beta^c \right) \quad (18)
 \end{aligned}$$

mixing between leptons and higgsinos

$$\begin{aligned}
& - \frac{\mu_{0a}}{2} L_a \tilde{\eta}' - \frac{\mu_{1a}}{2} L_a \tilde{\chi}' - \frac{\mu_2}{2} \tilde{\eta} \tilde{\chi}' - \frac{\mu_3}{2} \tilde{\chi} \tilde{\eta}' \\
& - \frac{\lambda_{2a}}{3} (L_a \tilde{\chi} \rho + \tilde{\rho} L_a \chi) - \frac{\lambda_{3a}}{3} (L_a \tilde{\eta} \rho + \tilde{\rho} L_a \eta) \\
& - \frac{\lambda_{4ab}}{3} L_a L_b \rho + h.c., \tag{19}
\end{aligned}$$

ψ^0

$$\begin{array}{cccccccccccc}
\nu_1 & \nu_2 & \nu_3 & \nu_1^c & \nu_2^c & \nu_3^c & -i\lambda_A^3 & -i\lambda_{X^0} & -i\lambda_{X^{0*}} & -i\lambda_A^8 & -i\lambda_B \\
\tilde{\eta}_1^0 & \tilde{\eta}_1'^0 & \tilde{\eta}_2^0 & \tilde{\eta}_2'^0 & \tilde{\chi}_1^0 & \tilde{\chi}_1'^0 & \tilde{\chi}_2^0 & \tilde{\chi}_2'^0 & \tilde{\rho}^0 & \tilde{\rho}'^0 &
\end{array} \tag{20}$$

$$-\frac{1}{2} (\Psi^0)^T Y^0 \Psi^0 + h.c. \tag{21}$$

$$\tilde{\chi}_i^0 = N_{ij} \Psi_j^0, \quad j = 1, \dots, 21. \tag{22}$$

$$Y^0 = \begin{pmatrix} (M_\nu)_{6 \times 6} & (M_{\nu N}) \\ (M_{\nu N}^T) & (M_N)_{15 \times 15} \end{pmatrix} \quad (23)$$

M_ν three Dirac eigenstates

$$0 \quad m_\nu \quad m_\nu \quad (24)$$

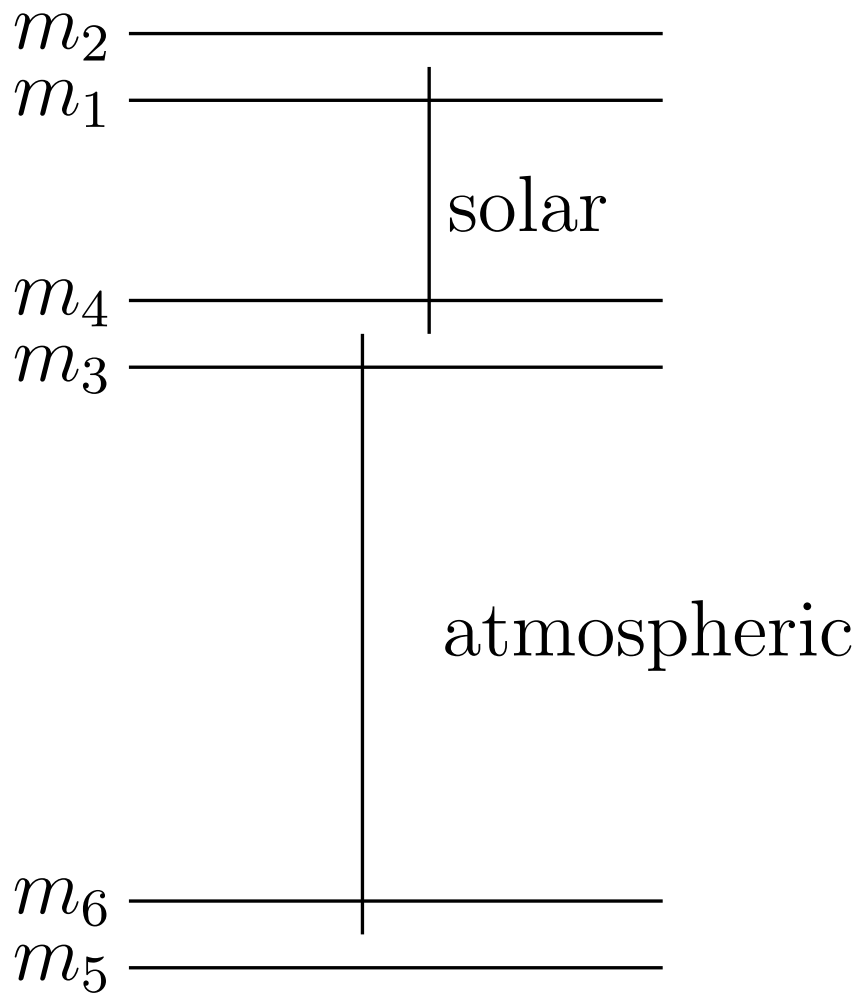
atmospheric neutrino mass difference

$$m_\nu = \frac{2u}{3} \sqrt{\lambda_{412}^2 + \lambda_{413}^2 + \lambda_{423}^2} \quad (25)$$

$$\begin{aligned} \lambda_{412} &= \lambda_{413} = \lambda_{423} = 2 \times 10^{-13}, \\ \lambda_{421} &= \lambda_{431} = \lambda_{432} = -2 \times 10^{-13}. \end{aligned} \quad (26)$$

$$\Delta m_{atm} \sim 5 \times 10^{-2} \text{ eV} \quad (27)$$

mixing terms turn on, this inverted spectrum will not only give rise to mass splitting between the two degenerate Dirac states, it will also split each Dirac pairs into two non-degenerate Majorana states



The inverted hierarchy mass pattern of neutrinos in the model

Conclusion

The famous relation for the R-parity in the MSSM has been generalized to this kind of the 3-3-1 models. In this case it relates to the new conserved charge \mathcal{L} . A simple mechanism for the mass generation of the neutrinos has been explored. We have showed that the model naturally gives rise to the neutrinos an inverted hierarchy mass pattern.

Acknowledgments

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