

Neutrino Theory: Some Recent Developments

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PREAMBLE

Neutrinos have mass and mix with one another.

PDG(2008):

$$\Delta m_{21}^2 = (8.0 \pm 0.3) \times 10^{-5} \text{ eV}^2,$$

$$\Delta m_{32}^2 = 1.9 \text{ to } 3.0 \times 10^{-3} \text{ eV}^2,$$

$$\sin^2(2\theta_{12}) = 0.86(+/-)(0.03/0.04), \quad \theta_{12} \simeq 34^\circ,$$

$$\sin^2(2\theta_{23}) > 0.92, \quad \text{best fit } \theta_{23} \simeq 45^\circ,$$

$$\sin^2(2\theta_{13}) < 0.19.$$

Brief History of Neutrino Tribimaximal Mixing

1978: Cabibbo and Wolfenstein conjectured independently that

$$U_{l\nu}^{CW} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix},$$

where $\omega = \exp(2\pi i/3) = -1/2 + i\sqrt{3}/2$. This should dispel the myth that everybody expected small mixing angles in the lepton sector as in the quark sector.

2002: Harrison/Perkins/Scott proposed the tribimaximal mixing matrix, i.e.

$$U_{l\nu}^{HPS} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix} \sim (\eta_8, \eta_1, \pi^0)$$

2004: I discovered the simple connection:

$$U_{l\nu}^{HPS} = (U_{l\nu}^{CW})^\dagger \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/\sqrt{2} & -1/\sqrt{2} \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & i \end{pmatrix}.$$

This means that if

$$\mathcal{M}_l = U_{l\nu}^{CW} \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix} (U_R^l)^\dagger$$

and \mathcal{M}_ν has 2 – 3 reflection symmetry, with zero 1 – 2 and 1 – 3 mixing, i.e.

$$\mathcal{M}_\nu = \begin{pmatrix} a + 2b & 0 & 0 \\ 0 & a - b & d \\ 0 & d & a - b \end{pmatrix},$$

$U_{l\nu}^{HPS}$ will be obtained, but **how?**.

Tribimaximal mixing means:

$$\theta_{13} = 0, \quad \sin^2 2\theta_{23} = 1, \quad \tan^2 \theta_{12} = 1/2.$$

In 2002 (when **HPS** proposed it), world data were not precise enough to test this idea.

In 2004 (when I derived it), **SNO** data implied $\tan^2 \theta_{12} = 0.40 \pm 0.05$, which was not so encouraging.

Then in 2005, **revised SNO** data obtained $\tan^2 \theta_{12} = 0.45 \pm 0.05$,
and **tribimaximal** became a household word,
unleashing a glut of papers.

Tetrahedral Symmetry A_4

For 3 families, we should look for a group with a 3 representation, the simplest of which is A_4 , the group of the **even** permutation of 4 objects.

class	n	h	χ_1	$\chi_{1'}$	$\chi_{1''}$	χ_3
C_1	1	1	1	1	1	3
C_2	4	3	1	ω	ω^2	0
C_3	4	3	1	ω^2	ω	0
C_4	3	2	1	1	1	-1

$$\omega = \exp(2\pi i/3) = -1/2 + i\sqrt{3}/2$$

Multiplication rule:

$$\begin{aligned} \underline{3} \times \underline{3} = & \underline{1}(\underline{11} + \underline{22} + \underline{33}) + \underline{1}'(\underline{11} + \omega^2\underline{22} + \omega\underline{33}) \\ & + \underline{1}''(\underline{11} + \omega\underline{22} + \omega^2\underline{33}) + \underline{3}(\underline{23}, \underline{31}, \underline{12}) + \underline{3}(\underline{32}, \underline{13}, \underline{21}). \end{aligned}$$

Note that $\underline{3} \times \underline{3} \times \underline{3} = \underline{1}$ is possible in A_4 ,

i.e. $a_1 b_2 c_3 + \text{permutations}$,

and $\underline{2} \times \underline{2} \times \underline{2} = \underline{1}$ is possible in S_3 ,

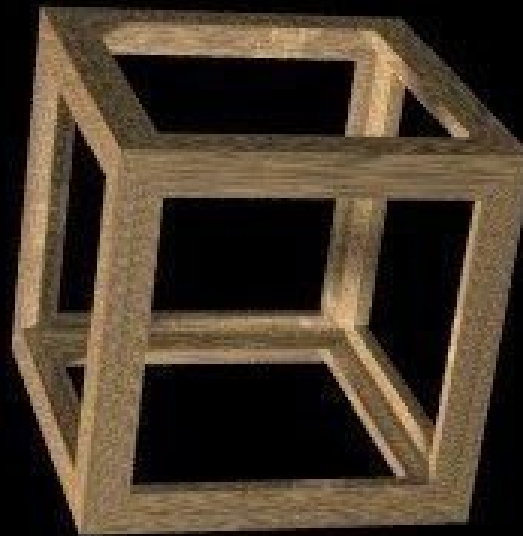
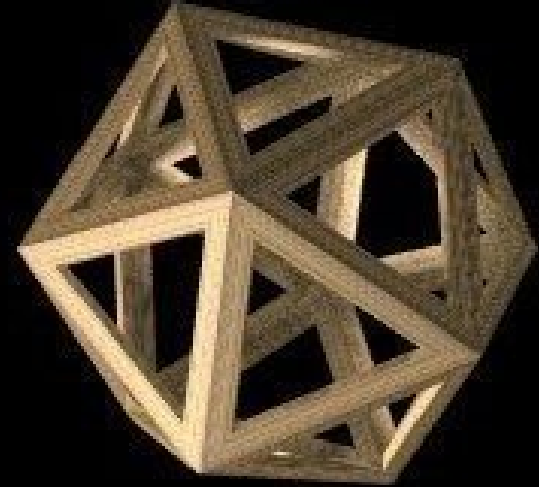
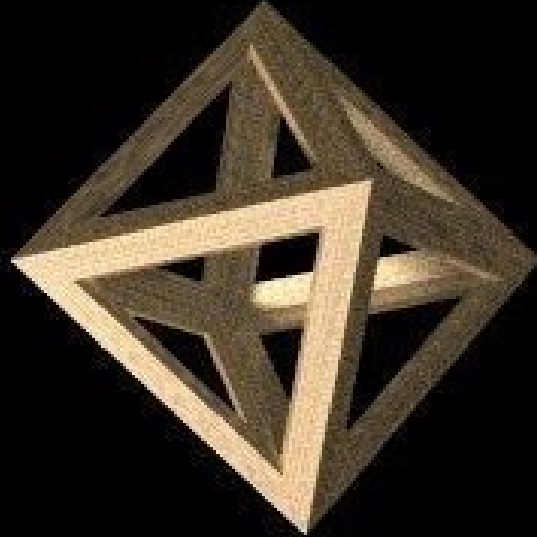
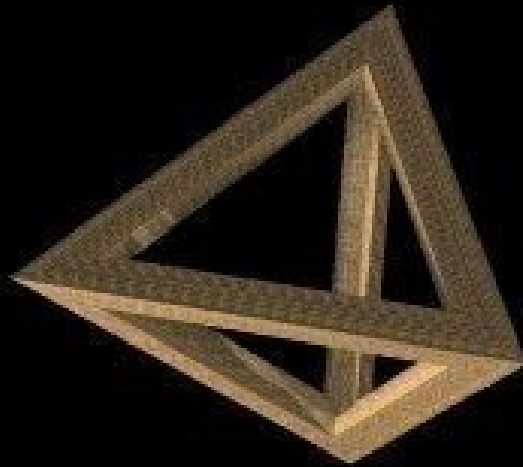
i.e. $a_1 b_1 c_1 + a_2 b_2 c_2$.

Perfect Three-Dimensional Geometric Solids:

solid	faces	vertices	Plato	group
tetrahedron	4	4	fire	A_4
octahedron	8	6	air	S_4
cube	6	8	earth	S_4
icosahedron	20	12	water	A_5
dodecahedron	12	20	quintessence	A_5

Amusingly, there are also 5 string theories in 10 dimensions: Type I is dual to Heterotic $SO(32)$, Type IIA is dual to Heterotic $E_8 \times E_8$, and Type IIB is self-dual.

THE
PLATONIC
SOLIDS



Two steps to tribimaximal mixing using A_4 :

(I) Choose (A) Ma/Rajasekaran(2001): $(\nu_i, l_i) \sim \underline{\underline{3}}$,

$l_i^c \sim \underline{1}, \underline{1}', \underline{1}''$, and $(\phi_i^0, \phi_i^-) \sim \underline{\underline{3}}$ with $v_1 = v_2 = v_3$, or

(B) Ma(2006): $(\nu_i, l_i) \sim \underline{\underline{3}}$, $l_i^c \sim \underline{\underline{3}}$, and $(\phi_i^0, \phi_i^-) \sim \underline{1}, \underline{\underline{3}}$,

with $v_1 = v_2 = v_3$. The diagonalization of \mathcal{M}_l yields

$U_{l\nu}^{CW}$ automatically, with arbitrary $m_{e,\mu,\tau}$. Here $A_4 \rightarrow Z_3$.

(II) In the neutrino sector, use $(\xi_i^{++}, \xi_i^+, \xi_i^0) \sim \underline{1}, \underline{\underline{3}}$, with

$u_2 = u_3 = 0$. Here $A_4 \rightarrow Z_2$. Ma(2004,2009);

Altarelli/Feruglio(2005):

$$\mathcal{M}_\nu = \begin{pmatrix} a & 0 & 0 \\ 0 & a & d \\ 0 & d & a \end{pmatrix}.$$

This is the simplest realization of **tribi**maximal mixing, with neutrino mass eigenvalues $a + d$, a , $-a + d$, allowing only normal hierarchy.

The **keys** to **tribi**maximal mixing are (1) the choice of symmetry, i.e. A_4 or S_4 , (2) the choice of lepton and Higgs representations, (3) residual symmetries Z_3 and Z_2 in two different sectors.

This is an example of how group theory alone could determine mixing angles, but leaves all masses free.

The Cabibbo angle = $\pi/14$ could come from $D_7 \rightarrow Z_2$:
[Lam(2007); Blum/Hagedorn/Lindner(2008).]

Seesaw Variations and the Nonunitarity of the Neutrino Mixing Matrix

With 1 doublet neutrino ν and 1 singlet neutrino N , their 2×2 mass matrix is the well-known

$$\mathcal{M}_{\nu N} = \begin{pmatrix} 0 & m_D \\ m_D & m_N \end{pmatrix},$$

resulting in the famous seesaw formula $m_\nu \simeq -m_D^2/m_N$. Hence $\nu - N$ mixing $\simeq m_D/m_N \simeq \sqrt{m_\nu/m_N} < 10^{-6}$, for $m_\nu < 1$ eV and $m_N > 1$ TeV.

Consider now 1 ν and 2 singlets: $N_{1,2}$. Their 3×3 mass matrix is then

$$\mathcal{M}_{\nu N} = \begin{pmatrix} 0 & m_D & 0 \\ m_D & m_1 & m_N \\ 0 & m_N & m_2 \end{pmatrix},$$

resulting in $m_\nu \simeq m_D^2 m_2 / m_N^2$. Since the limit $m_1 = 0$ and $m_2 = 0$ corresponds to lepton number conservation ($L = 1$ for ν and N_2 , $L = -1$ for N_1), their smallness is **natural** \Rightarrow the inverse seesaw [Mohapatra/Valle(1986)]. Here $\nu - N_1$ mixing is still small, but $\nu - N_2$ mixing $\simeq m_D / m_N$ may be big \Rightarrow observable **nonunitarity** of $U_{l\nu}$.

Consider next $\nu_{1,2}$ and $N_{1,2}$, where

$$\mathcal{M}_D = \begin{pmatrix} a_1 b_1 & a_1 b_2 \\ a_2 b_1 & a_2 b_2 \end{pmatrix} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} (b_1 \quad b_2),$$

and $\mathcal{M}_N = \text{diag}(M'_1, M'_2)$. In that case, the arbitrary imposed condition $b_1^2/M'_1 + b_2^2/M'_2 = 0$ renders all two light neutrinos massless, and yet $\nu - N$ mixing may be large, because $a_i b_j$ need not be small. Small deviations from this texture will allow small neutrino masses and retain the large $\nu - N$ mixing. This is an active topic of study: see e.g. [Pilaftsis\(2005\)](#); [Kersten/Smirnov\(2007\)](#).

He/Ma(2009): To understand the mechanism and symmetry of the texture hypothesis, change the neutrino basis to

$$\mathcal{M}_D = \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix}, \quad \mathcal{M}_N = \begin{pmatrix} M_1 & M_3 \\ M_3 & M_2 \end{pmatrix}.$$

Then the condition is $m_1 = M_1 = 0$, so that ν_1 and $\nu'_2 = (M_3\nu_2 - m_2 N_1) / \sqrt{M_3^2 + m_2^2}$ are massless, showing how **large mixing actually occurs**, i.e. through the **inverse seesaw mechanism**. However, lepton number conservation would not only forbid M_1 but also M_2 , which is in fact arbitrary here. Where is the **symmetry** which does this?

Let $\nu_{1,2}$, $N_{1,2}$ have $L = 1, 1, 3, -1$.

Add the usual Higgs doublet (ϕ_1^+, ϕ_1^0) with $L = 0$ and the Higgs singlet χ_2 with $L = 2$. Then m_2 comes from $\langle \phi_1^0 \rangle$, M_2 from $\langle \chi_2 \rangle$, and M_3 from $\langle \chi_2^\dagger \rangle$.

$m_1 = 0$ at tree level, because there is no Higgs doublet with $L = -4$, and $M_1 = 0$ at tree level, because there is no Higgs singlet with $L = \pm 6$.

In **one loop**, M_1 will be induced, thus giving ν_2' an inverse seesaw mass $= M_1 m_2^2 / M_3^2$. Once ν_2 is massive, ν_1 also gets a two-loop radiative mass from the exchange of two W 's [[Babu/Ma\(1988\)](#)].

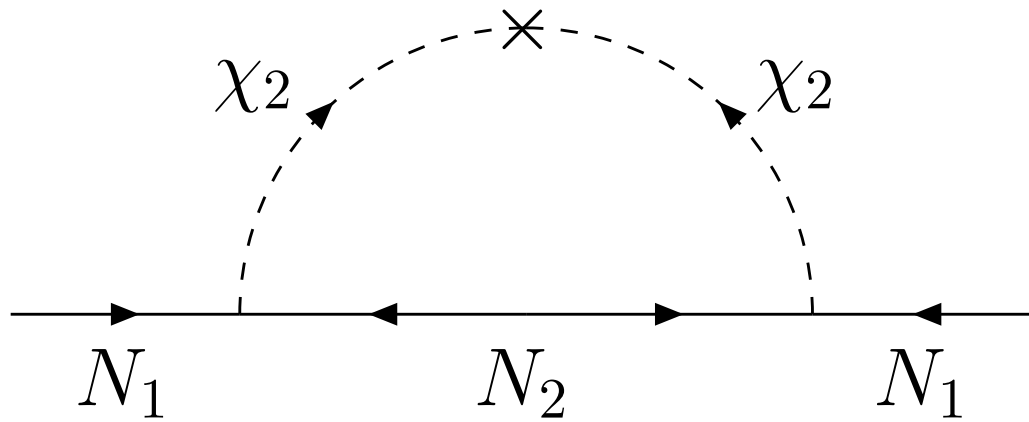


Figure 1: One-loop generation of M_1 .

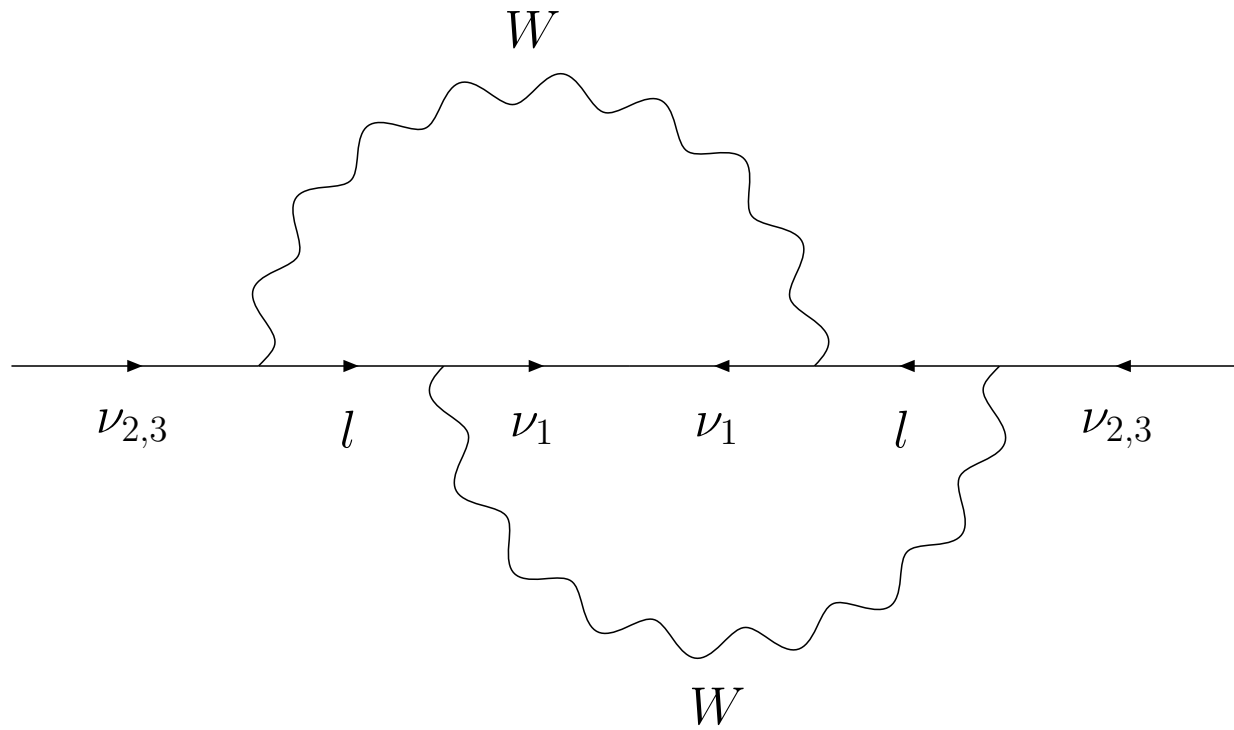


Figure 1: Two- W generation of neutrino mass.

Radiative Seesaw and Dark Matter: Scotogenic Neutrino Mass

Ma(2006): Add to the Standard Model (SM) a second scalar doublet (η^+, η^0) and 3 neutral singlet Majorana fermions $N_{1,2,3}$ which are odd under an exactly conserved Z_2 , with all SM particles even. Hence $\nu N \phi^0$ is forbidden and $\nu N \eta^0$ is allowed, but $\langle \eta^0 \rangle = 0$. Neutrino mass is generated in one loop, i.e. **scotogenic**, being caused by darkness. Here, η_R^0 is a **dark-matter** candidate, later studied by [Barbieri/Hall/Rychkov(2006)]. They call η the inert Higgs doublet. I call it the **dark scalar doublet**.

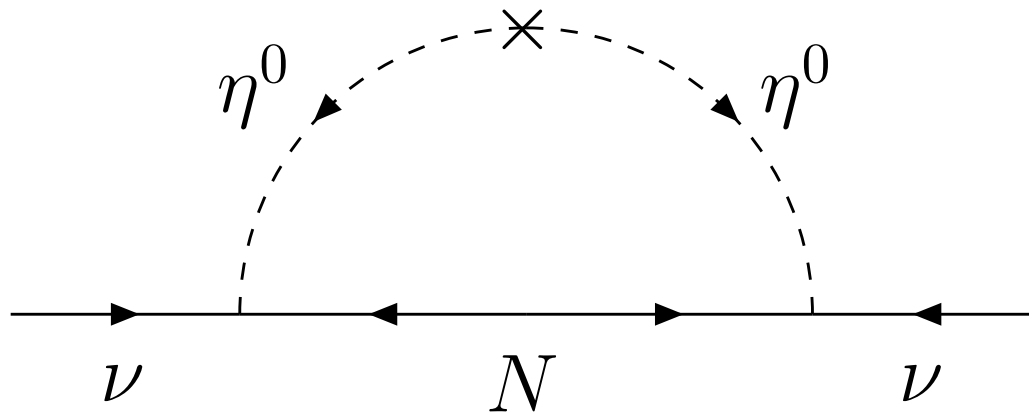
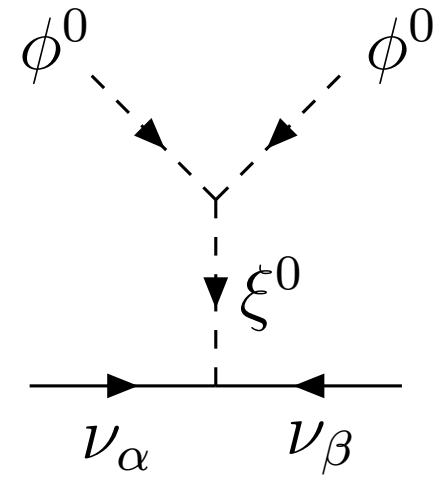
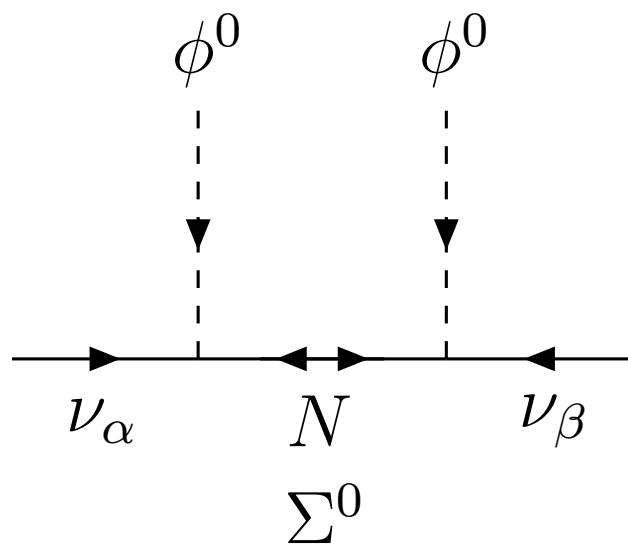


Figure 1: One-loop m_ν from Z_2 dark matter.



The mass splitting between η_R^0 and η_I^0 comes from the Z_2 allowed term $(\lambda_5/2)(\Phi^\dagger\eta)^2 + H.c.$

$$(\mathcal{M}_\nu)_{\alpha\beta} = \sum_i \frac{h_{\alpha i} h_{\beta i} M_i}{16\pi^2} [f(M_i^2/m_R^2) - f(M_i^2/m_I^2)],$$

where $f(x) = -\ln x/(1-x)$.

Let $m_R^2 - m_I^2 = 2\lambda_5 v^2 \ll m_0^2 = (m_R^2 + m_I^2)/2$, then

$$(\mathcal{M}_\nu)_{\alpha\beta} = \sum_i \frac{h_{\alpha i} h_{\beta i}}{M_i} I(M_i^2/m_0^2),$$

$$I(x) = \frac{\lambda_5 v^2}{8\pi^2} \left(\frac{x}{1-x} \right) \left[1 + \frac{x \ln x}{1-x} \right].$$

For $x_i \gg 1$, i.e. N_i very heavy,

$$(\mathcal{M}_\nu)_{\alpha\beta} = \frac{\lambda_5 v^2}{8\pi^2} \sum_i \frac{h_{\alpha i} h_{\beta i}}{M_i} [\ln x_i - 1]$$

instead of the canonical seesaw $v^2 \sum_i h_{\alpha i} h_{\beta i} / M_i$.

In **leptogenesis**, the lightest M_i may then be much below the **Davidson-Ibarra** bound of about 10^9 GeV, thus avoiding a potential conflict of **gravitino** overproduction and thermal **leptogenesis** if **supersymmetry** is considered.

Supersymmetric $SU(5)$ Completion:

Gauge-coupling unification in the **MSSM** is preserved by complete $SU(5)$ multiplets.

Ma(2007): Add $\underline{5} = h(3, 1, -1/3) + (\eta_2^+, \eta_2^0)(1, 2, 1/2)$,
 $\underline{5}^* = h^c(3^*, 1, 1/3) + (\eta_1^0, \eta_1^-)(1, 2, -1/2)$,
and singlet superfields N and χ .

Under $Z_2^{(1)} \times Z_2^{(2)}$, let

$$(u, d), u^c, d^c, (\nu, e), e^c \sim (-, +);$$

$$(\phi_1^0, \phi_1^-), (\phi_2^+, \phi_2^0) \sim (+, +);$$

$$h, h^c, (\eta_1^0, \eta_1^-), (\eta_2^+, \eta_2^0) \sim (+, -);$$

$$N \sim (-, -); \chi \sim (+, -).$$

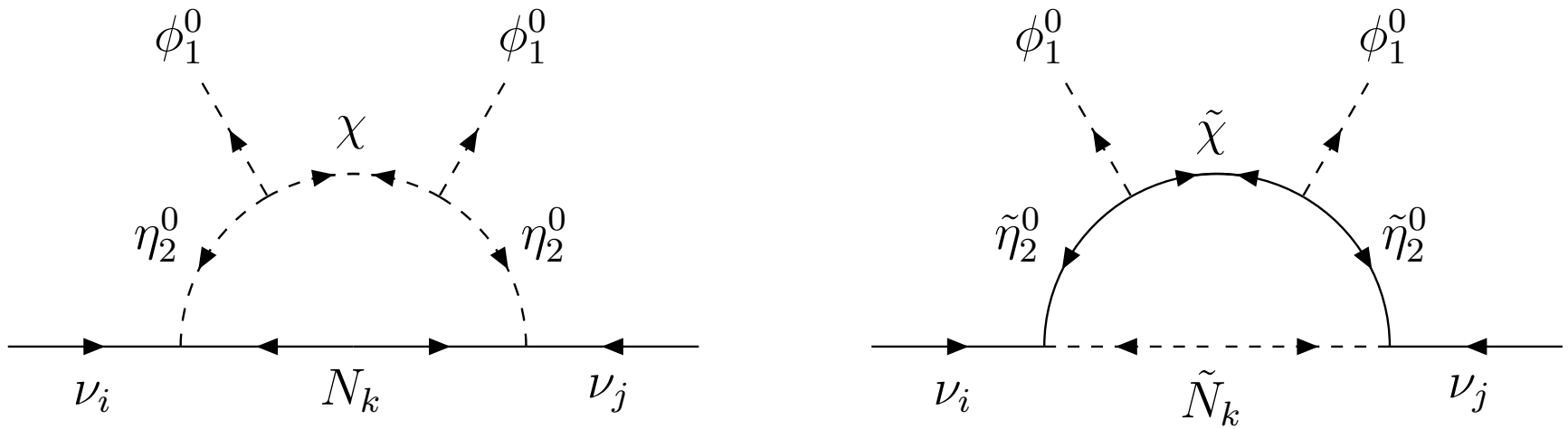


Figure 1: One-loop supersymmetric neutrino mass.

The first $Z_2^{(1)}$ yields the usual R parity of the **MSSM**.

The second $Z_2^{(2)}$ allows **scotogenic** neutrino mass through the terms $(\nu\eta_2^0 - e\eta_2^+)N$, $\phi_1^0\eta_2^0\chi$, NN , and $\chi\chi$.

This extension of the **DSDM** is safe for proton decay because the would-be mediators h, h^c are odd under $Z_2^{(2)}$.

It also predicts the strong production of $h\bar{h}$ at the Large Hadron collider, with $h \rightarrow de^- \eta_2^+$ or $de^+ \eta_2^-$.

Thus a possible unique signature of this model is the appearance of **same-sign** dileptons plus quark jets plus missing energy.

At least **two** out of the following **three** particles are dark-matter candidates:

- (1) the usual lightest neutralino of the **MSSM** with $(R, Z_2^{(2)}) = (-, +)$,
- (2) the lightest exotic neutral particle with $(+, -)$,
- (3) and that with $(-, -)$.

The dark matter of the Universe may not be all the same, as most people have taken for granted! For a general discussion, see **Cao/Ma/Wudka/Yuan, arXiv:0711.3881.**

Krauss/Nasri/Trodden(2003):

Neutrino mass may be obtained in 3 loops, with neutral singlet fermions N and charged scalar S_2^+ having odd Z_2 . This was the first proposal that N could be dark matter. However, since lNS_2^+ is the only interaction involving N , it has to be rather big to have the correct annihilation cross section for the presumed dark-matter relic density. Flavor-changing radiative decays such as $\mu \rightarrow e\gamma$ are then too big, without extreme fine tuning. In the one-loop **scotogenic** case, this constraint is relaxed, because η_R^0 is available for dark matter.

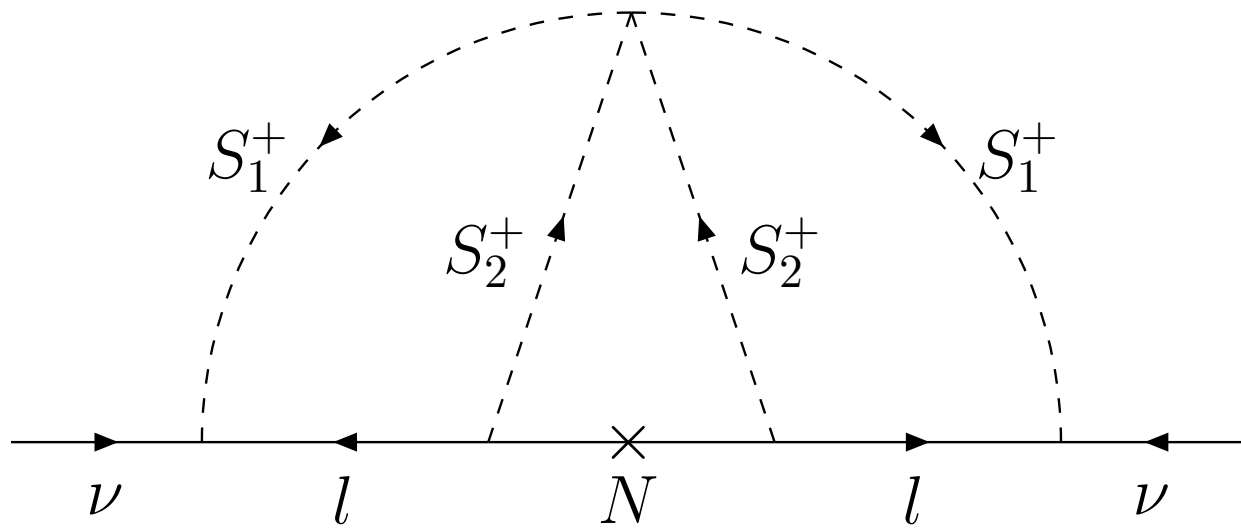


Figure 1: Three-loop neutrino mass (Krauss/Nasri/Trodden).

Aoki/Kanemura/Seto(2009):

Instead of two charged singlets, they propose two Higgs doublets and a neutral singlet η^0 (40 – 65 GeV), so that the latter can be dark matter, instead of N (3 TeV).

This model allows for electroweak baryogenesis, coming from a first-order phase transition in the Higgs potential.

At the LHC, the η^0 is hard to produce and detect because it is a singlet, whereas η_R^0 and η_I^0 of the one-loop **scotogenic** model are produced by the Z boson, with the subsequent decay $\eta_I^0 \rightarrow \eta_R^0 l^+ l^-$ as a possible signature.

[Cao/Ma/Rajasekaran(2007)]

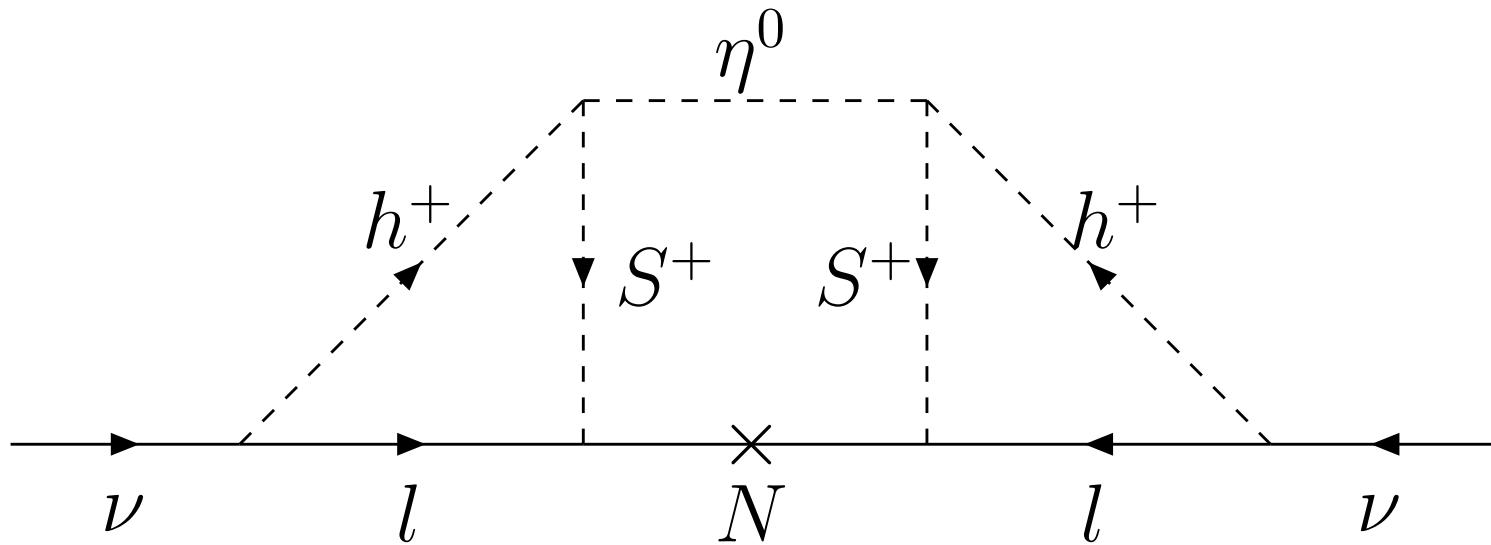


Figure 1: Three-loop neutrino mass (Aoki/Kanemura/Seto).

Concluding Remarks

- With the application of the non-Abelian discrete symmetry A_4 , a plausible theoretical understanding of the tribimaximal form of the neutrino mixing matrix has been achieved.
- Seesaw variants at the TeV scale may allow this mechanism (in its inverse or linear manifestation) to be observable through the nonunitarity of the 3×3 neutrino mixing matrix, as well as flavor changing leptonic interactions.

- Dark matter may be the origin of radiative neutrino mass. This **scotogenic** mechanism may be implemented in a number of different models, and be observable also at the TeV scale. A complete supersymmetric $SU(5)$ version also exists with gauge-coupling unification.
- Other recent topics in neutrino theory, such as Type III seesaw, using a Majorana fermion triplet $(\Sigma^+, \Sigma^0, \Sigma^-)$, and small Dirac and pseudo-Dirac neutrino masses, are not covered in this talk, but are being actively pursued.
- Neutrino theory marches on, but what we really need are corroborating **data!**