Supersymmetry and Extra Vector-like Generation

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Abstract

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Phys. Rev. D 80 (2009) 035004 (arXiv:0907.3011)

5th ICFP, Hanoi, September 24-30, 2009

I. Introduction

Consider SUSY with the 4th generation,

MSSM: 3 generation matters $\oplus \underbrace{H_d + H_u}_{two\ \ Higgs\ \ doublets}$

$$\begin{pmatrix} L \\ E^c \\ Q \\ U^c \\ D^c \end{pmatrix}_1 , \begin{pmatrix} L \\ E^c \\ Q \\ U^c \\ D^c \end{pmatrix}_2 , \begin{pmatrix} L \\ E^c \\ Q \\ U^c \\ D^c \end{pmatrix}_3 , \underbrace{\begin{pmatrix} L \\ E^c \\ Q \\ U^c \\ D^c \end{pmatrix}_4 , \begin{pmatrix} H_u \\ E^c_H \\ Q_H \\ U^H \\ D^H \end{pmatrix}}_{opposite\ \ SU(3)\ \&\ U(1)}$$

$$L_4 \equiv H_d$$

Two comments:

1. H_d, $H_u \rightarrow$ **chiral matter (like 3 generations)**

μ-problem?

2. Worse than SUSY GUT;

can be tested at LHC.

II. Model

Two extra generations (one vector-like)

New particles under $\mathbf{SU(2)}_L\times\mathbf{U(1)}_Y\times\mathbf{SU(3)}_c$ and baryon number:

$$L_m(2,-1,1,0)\,,E^c_m(1,2,1,0)\,,Q_m(2,\tfrac{1}{3},3,\tfrac{1}{3})\,,U^c_m(1,-\tfrac{4}{3},\bar{3},-\tfrac{1}{3})\,,D^c_m(1,\tfrac{2}{3},\bar{3},-\tfrac{1}{3})\,,$$

$$H_u(2,1,1,0)\,,E^c_H(1,-2,1,0)\,,Q_H(2,-\tfrac{1}{3},\bar{3},-\tfrac{1}{3})\,,U^c_H(1,\tfrac{4}{3},3,\tfrac{1}{3})\,,D^c_H(1,-\tfrac{2}{3},3,\tfrac{1}{3})\,,$$

where $m = 1-4$.

$(H_u,\ E^c_H,\ Q_H,\ U^c_H,\ D^c_H)$ — **anti-particle rep. of other 4 generations.**

Baryon number conservation assumed.

$$\begin{aligned}\mathcal{W} \;=\;& \mu_m L_m H_u + \mu^e_m E^c_m E^c_H + \mu^Q_m Q_m Q_H + \mu^U_m U^c_m U^c_H + \mu^D_m D^c_m D^c_H + \lambda_{lmn} L_l L_m E^c_n \\ & \lambda'_{lmn} Q_l L_m D^c_n + y_{mn} Q_m H_u U^c_n + y'_m Q_H L_m U^c_H + \tilde{y}_m E^c_H D^c_m U^c_H + \tilde{y}_{mn} E^c_m D^c_H U^c_n\,,\end{aligned} \tag{1}$$

μ_m - mass parameters
$\lambda^{(\prime)}$, $y^{(\prime)}$ and $\tilde{y}^{(\prime)}$'s - coefficients.

Field redefinition:

$$H_d \equiv \frac{\mu_m}{\mu} L_m \,, \;\; E_4^c \equiv \frac{\mu_m^e}{\mu^e} E_m^c \,, \;\; Q_4 \equiv \frac{\mu_m^Q}{\mu^Q} Q_m \,, \;\; U_4^c \equiv \frac{\mu_m^U}{\mu^U} U_m^c \,, \;\; D_4^c \equiv \frac{\mu_m^D}{\mu^D} D_m^c \,,$$

where

$$\mu \equiv \sqrt{\sum_{m=1}^{4} |\mu_m|^2} \,, \;\; \mu^e \equiv \sqrt{\sum_{m=1}^{4} |\mu_m^e|^2} \,, \;\; \mu^Q \equiv \sqrt{\sum_{m=1}^{4} |\mu_m^Q|^2} \,, \;\; \mu^U \equiv \sqrt{\sum_{m=1}^{4} |\mu_m^U|^2} \,, \;\; \mu^D \equiv \sqrt{\sum_{m=1}^{4} |\mu_m^D|^2} \,,$$

the superpotential is

$$\begin{aligned} \mathcal{W} \;=\; & \mu H_d H_u + \mu^e E_4^c E_H^c + \mu^Q Q_4 Q_H + \mu^U U_4^c U_H^c + \mu^D D_4^c D_H^c + y_{ij}^l L_i H_d E_j^c + y_{ij}^d Q_i H_d D_j^c \\ & + y_{ij}^u Q_i H_u U_j^c + \lambda_{ijk} L_i L_j E_k^c + \lambda'_{ijk} Q_i L_j D_k^c + \lambda_{ij}^E L_i L_j E_4^c + y_i^E L_i H_d E_4^c + \lambda_{ij}^Q Q_4 L_i D_j^c \\ & + y_i^{Q\prime} Q_4 H_d D_i^c + \lambda_{ij}^D Q_i L_j D_4^c + y_i^D Q_i H_d D_4^c + \lambda_i^{QD} Q_4 L_i D_4^c + y^{QD} Q_4 H_d D_4^c + y_i^U Q_i H_u U_4^c \\ & + y_i^Q Q_4 H_u U_i^c + y^{QU} Q_4 H_u U_4^c + \lambda'_i Q_H L_i U_H^c + y Q_H H_d U_H^c + \tilde{\lambda}_i E_H^c D_i^c U_H^c + \tilde{\lambda} E_H^c D_4^c U_H^c \\ & + \tilde{\lambda}_{ij} E_i^c D_H^c U_j^c + \tilde{\lambda}_i^U E_i^c D_H^c U_4^c + \tilde{\lambda}_i^E E_4^c D_H^c U_i^c + \tilde{\lambda}^{EU} E_4^c D_H^c U_4^c \,, \end{aligned} \tag{2}$$

Generally,

$$\begin{aligned} L_m = c_{mi} L_i + c_{m4} H_d \,, \quad E_m^c = c_{mi}^e E_i^c + c_{m4}^e E_4^c \,, \quad Q_m = c_{mi}^Q Q_i + c_{m4}^Q Q_4 \,, \\ U_m^c = c_{mi}^U U_i^c + c_{m4}^U U_4^c \,, \quad D_m^c = c_{mi}^D D_i^c + c_{m4}^D D_4^c \,, \end{aligned} \tag{3}$$

and the coefficients are

$$\begin{gathered} y_{ij}^l = 2\lambda_{lmn} c_{li} c_{m4} c_{nj}^e \,, \quad y_{ij}^d = \lambda'_{lmn} c_{li}^Q c_{m4} c_{nj}^D \,, \quad y_{ij}^u = y_{mn} c_{mi}^Q c_{nj}^U \,, \quad \lambda_{ijk} = \lambda_{lmn} c_{li} c_{mj} c_{nk}^e \,, \\ \lambda'_{ijk} = \lambda'_{lmn} c_{li}^Q c_{mj} c_{nk}^D \,, \quad \lambda_{ij}^E = \lambda_{lmn} c_{li} c_{mj} c_{n4}^e \,, \quad y_i^E = 2\lambda_{lmn} c_{li} c_{m4} c_{n4}^e \,, \quad \lambda_{ij}^Q = \lambda'_{lmn} c_{l4}^Q c_{mi} c_{nj}^D \,, \\ y_i^{Q\prime} = \lambda'_{lmn} c_{l4}^Q c_{m4} c_{ni}^D \,, \lambda_{ij}^D = \lambda'_{lmn} c_{li}^Q c_{mj} c_{n4}^D \,, \quad y_i^D = \lambda'_{lmn} c_{li}^Q c_{m4} c_{n4}^D \,, \quad \lambda_i^{QD} = \lambda'_{lmn} c_{l4}^Q c_{mi} c_{n4}^D \,, \\ y^{QD} = \lambda'_{lmn} c_{l4}^Q c_{m4} c_{n4}^D \,, \quad y_i^U = y_{mn} c_{mi}^Q c_{n4}^U \,, \quad y_i^Q = y_{mn} c_{m4}^Q c_{ni}^U \,, \quad y^{QU} = y_{mn} c_{m4}^Q c_{n4}^U \,, \\ \lambda'_i = y'_m c_{mi} \,, \quad y = y'_m c_{m4} \,, \quad \tilde{\lambda}_i = \tilde{y}_m c_{mi}^D \,, \quad \tilde{\lambda} = \tilde{y}_m c_{m4}^D \,, \\ \tilde{\lambda}_{ij} = \tilde{y}_{mn} c_{mi}^e c_{nj}^U \,, \quad \tilde{\lambda}_i^U = \tilde{y}_{mn} c_{mi}^e c_{n4}^U \,, \quad \tilde{\lambda}_i^E = \tilde{y}_{mn} c_{m4}^e c_{ni}^U \,, \quad \tilde{\lambda}^{EU} = \tilde{y}_{mn} c_{m4}^e c_{n4}^U \,. \end{gathered} \tag{4}$$

Dirac mass terms!

One of the 4 generations, namely the 4th,

massive!

$$H_d H_u, \quad E_4^c E_H^c, \quad Q_4 Q_H, \ ......$$

$$\Uparrow \qquad\qquad \Uparrow \qquad\qquad \Uparrow$$

(note the 4th generation)

4th generation neutrino massive - higgsino!

Many new interactions.

II.-1 SUSY breaking

Soft masses

$$
\begin{aligned}
-\mathcal{L} \supset\ & M^2 \tilde{L}_m^\dagger \tilde{L}_m + M_h^2 h_u^\dagger h_u + M_E^2 \tilde{E}_m^{c\dagger} \tilde{E}_m^c + M_Q^2 \tilde{Q}_m^\dagger \tilde{Q}_m + M_U^2 \tilde{U}_m^{c\dagger} \tilde{U}_m^c + M_D^2 \tilde{D}_m^{c\dagger} \tilde{D}_m^c \\
& + M_{EH}^2 \tilde{E}_H^{c*} \tilde{E}_H^c + M_{QH}^2 \tilde{Q}_H^\dagger \tilde{Q}_H + M_{UH}^2 \tilde{U}_H^{c*} \tilde{U}_H^c + M_{DH}^2 \tilde{D}_H^{c*} \tilde{D}_H^c \\
& + (B\mu_m \tilde{L}_m h_u + B^e \mu_m^e \tilde{E}_m^c \tilde{E}_H^c + B^Q \mu_m^Q \tilde{Q}_m \tilde{Q}_H + B^U \mu_m^U \tilde{U}_m^c \tilde{U}_H^c + B^D \mu_m^D \tilde{D}_m^c \tilde{D}_H^c + h.c.
\end{aligned}
\tag{5}
$$

Universality: $M^2_{(h,E,Q,U,D)}$ and $B^{e,Q,U,D}$

not depend on the sub-script m.

$$
\begin{aligned}
-\mathcal{L} \supset\ & M^2 \tilde{L}_i^\dagger \tilde{L}_i + M^2 h_d^\dagger h_d + M_h^2 h_u^\dagger h_u + M_E^2 \tilde{E}_m^{c\dagger} \tilde{E}_m^c + M_Q^2 \tilde{Q}_m^\dagger \tilde{Q}_m + M_U^2 \tilde{U}_m^{c\dagger} \tilde{U}_m^c + M_D^2 \tilde{D}_m^{c\dagger} \tilde{D}_m^c \\
& + M_{EH}^2 \tilde{E}_H^{c*} \tilde{E}_H^c + M_{QH}^2 \tilde{Q}_H^\dagger \tilde{Q}_H + M_{UH}^2 \tilde{U}_H^{c*} \tilde{U}_H^c + M_{DH}^2 \tilde{D}_H^{c*} \tilde{D}_H^c \\
& + (B\mu h_d h_u + B^e \mu^e \tilde{E}_4^c \tilde{E}_H^c + B^Q \mu^Q \tilde{Q}_4 \tilde{Q}_H + B^U \mu^U \tilde{U}_4^c \tilde{U}_H^c + B^D \mu^D \tilde{D}_4^c \tilde{D}_H^c + h.c.)\,.
\end{aligned}
\tag{6}
$$

Soft trilinear: alighnment assumed.

II.-2 Gauge symmetry breaking

New $B\mu$ terms might complicate the gauge symmetry breaking analysis.

The larger the coefficients of quartic terms, the more difficult the gauge symmetry breaking is.

$\not{E}W$ is easier than $\not{c}olor$,

SU(2)$_L$×U(1)$_Y$: $V \supset \frac{g^2+g'^2}{8}\left(h_u^\dagger h_u - h_d^\dagger h_d\right)^2 + \ldots\ldots,$

Purely U(1)$_Y$: $V \supset \frac{g'^2}{2}\left(\tilde{E}_H^{c*}\tilde{E}_H^c - \tilde{E}_4^{c*}\tilde{E}_4^c\right)^2 + \ldots\ldots.$

Note $g'^2/2 \sim (g^2+g'^2)/8$ **numerically.**

Correct EWSB requires:

$$\begin{aligned}
(M^2+\mu^2)(M_h^2+\mu^2) &< |B\mu|^2\,, \\
(M_X^2+\mu^{X2})(M_{XH}^2+\mu^{X2}) &> |B^X\mu^X|^2 \quad \mathrm{for X = e, Q, U, D}\,,
\end{aligned} \tag{7}$$

with the ordinary one $M^2 + M_U^2 + 2\mu^2 + 2B\mu > 0$**.**

Higgs sector (including higgsino/gauginos) = MSSM

II.-3 Fermion spectra

Leptons:

$$\mathcal{L} \supset -\left(e_i^-, e_H^c\right) \mathcal{M}^l \begin{pmatrix} e_j^c \\ e_4^c \end{pmatrix}, \tag{8}$$

4×4 charged lepton mass matrix,

$$\mathcal{M}^l = \begin{pmatrix} m_{ij}^l & m_{i4}^l \\ 0 & \mu^e \end{pmatrix}, \tag{9}$$

$$\begin{aligned} m_\tau &\simeq \sqrt{|m_{33}^l|^2 - |m_{33}^l||m_{34}^l|}\,, \\ M_l &\simeq |\mu^e|\,. \end{aligned} \tag{10}$$

The unitary matrix diagonalizing $\mathcal{M}^l \mathcal{M}^{l\dagger}$ is then

$$\begin{pmatrix} 1 & -\left(1 + \dfrac{|m_{33}^l|}{|m_{34}^l|}\right) \dfrac{m_{34}^{l*}}{\mu^{e*}} \\ \dfrac{m_{34}^l}{\mu^e} & 1 \end{pmatrix} + \mathcal{O}\left(\frac{m_\tau}{\mu^e}\right)^2. \tag{11}$$

down-type quarks:

$$\mathcal{L} \supset -\left(q_i^b, q_4^b, d_H^c\right) \mathcal{M}^d \begin{pmatrix} d_j^c \\ d_4^c \\ q_H^t \end{pmatrix}, \tag{12}$$

$$\mathcal{M}^d = \begin{pmatrix} m_{ij}^d & m_{i4}^d & 0 \\ m_{4j}^d & m_{44}^d & -\mu^Q \\ 0 & -\mu^D & 0 \end{pmatrix}, \tag{13}$$

The mass matrix is diagonalized by unitary matrices U^d and V^d,

$$U^{d\dagger}\mathcal{M}^d V^d = \begin{pmatrix} m_b & 0 & 0 \\ 0 & \mu^Q & 0 \\ 0 & 0 & \mu^D \end{pmatrix} \tag{14}$$

where

$$U^d = \begin{pmatrix} 1 & 0 & -\frac{m_{34}^d}{\mu^D} \\ 0 & 1 & -\frac{|m_{34}^d|^2}{\mu^D m_{44}^{d*}} \\ -\frac{m_{34}^{d*}}{\mu^{D*}} & \frac{\mu^D m_{44}^{d*}}{|\mu^D|^2 - |\mu^Q|^2} & 1 \end{pmatrix} + \mathcal{O}\left(\frac{m_b}{\mu^D}\right)^2, \tag{15}$$

and

$$V^d = \begin{pmatrix} 1 & -\frac{m_{43}^{d*}}{\mu^{Q*}} & 0 \\ 0 & \frac{|m_{43}^d|^2}{\mu^{Q*} m_{44}^d} & 1 \\ \frac{m_{43}^d}{\mu^Q} & 1 & \frac{\mu^{Q*} m_{44}^d}{|\mu^Q|^2 - |\mu^D|^2} \end{pmatrix} + \mathcal{O}\left(\frac{m_b}{\mu^D}\right)^2. \tag{16}$$

Up quarks ....(similar)

III. Phenomenology

LEP: Extra generation particles should be heavier than 100 GeV.

Tevatron: Extra generation quarks are heavier than 270 GeV.

Electroweak precision measurement:

$$S \simeq T \simeq \left(\frac{m_t}{\mu^X}\right)^2 . \tag{17}$$

The effect of the extra generation can be small enough $\leq \left(m_t/\mu^X\right)^2 \simeq (1-10)\%$ if we take $\mu^X \simeq 500$ GeV -1 TeV.

Unitarity of the 3×3 CKM: important extra generations mix with ordinary three chiral generations which necessarily break the unitarity of the CKM mixing matrix. Unitarity violation is about $(m_{i4}^{d(u)}/\mu^{D(U)})^2$. This μ^X dependence is generally expected in the case of extra vector-like generations. Hierarchical or small mixing masses $m_{i4}^{d(u)}$ can easily make the CKM matrix approximately unitary within errors. For an example, $(m_{14}^{d(u)}/\mu^X)^2 \le 10^{-3}$. Assuming only the third generation mixes with extra generations, the constraint is still loose, $(m_{34}^{d(u)}/\mu^{D(U)})^2 \le 0.39$. The quantity $m_{34}^{d(u)}$ is at most about m_t. This gives that the parameter $\mu^{D,U} \ge 280$ GeV.

There are new phases in fermion mixing matrices. However, these new matrix elements are of order of $\left(m_t/\mu^X\right)^2$ at most. So new CP violation effects are generally suppressed.

At LHC, in addition to the (super) particle content of MSSM, it predicts following vector-like particles: one lepton singlet (E_4^c, E_H^c), one quark doublet (Q_4, Q_H), one up-type quark singlet (U_4^c, U_H^c) and one down-type quark singlet (D_4^c, D_H^c), but there is no extra doublet leptons which are already identified as the Higgs doublets.

IV. Discussion

Try to put H_d and H_u into chiral families.

New particles are expected at LHC.

First generation — ordinary matter,

Second generation — fermion mixing,

Third generation — CP violation,

Fourth and fifth (the vector one) — EWSB.

Baryon number conservation can be a discrete gauge symmetry.

……

Detailed flavor physics of this model is under consideration.