

Flavor Physics in the Littlest Higgs model with T-parity

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T. Goto, Y. Okada and Y. Yamamoto, Phys. Lett. B **670** (2009) 378 [arXiv:0809.4753] $+\alpha$.

Introduction

Little Higgs models attract interests in recent years.

- Possible solution to *little hierarchy problem*.
 - ▷ Higgs boson (SM) mass $\sim 100\text{GeV}$ vs. NP scale $\Lambda \gtrsim O(\text{TeV})$.
 $\Rightarrow O(\Lambda^2)$ one-loop correction to the Higgs boson mass².
- Basic idea: Higgs bosons are assumed to be pseudo Nambu-Goldstone bosons of a spontaneous global symmetry breaking.
- One-loop correction to the Higgs boson mass is suppressed by *collective symmetry breaking*.

Littlest Higgs model (Arkani-Hamed *et al.*, 2002)

- An implementation based on $SU(5)/SO(5)$ nonlinear sigma model.
- Electroweak precision constraints
 $\Rightarrow SU(5) \rightarrow SO(5)$ SSB scale $\gtrsim 4\text{TeV}$ (Csaki *et al.*).
 - ▷ Little hierarchy problem revives.
- Discrete Z_2 symmetry “**T-parity**” helps (Cheng & Low, 2003).

\Rightarrow LHT is phenomenologically viable.

Introduction: flavor physics in LHT

LHT provides new particles (heavy gauge bosons and quarks/leptons) and [new flavor mixings](#).

⇒ Various signals in flavor observables expected and studied.

- Blanke *et al.* (2006–2007,2009), Hubisz *et al.* (2006), Choudhury *et al.* (2007), ...

We have recalculated flavor changing amplitudes in LHT.

- A missing piece found, that cancels previously reported UV sensitivity (cut-off dependence).
- Applied to $K \rightarrow \pi \nu \bar{\nu}$ and lepton flavor violations.

Contents in the following:

- The model: LHT,
- Flavor changing Z penguin amplitude,
- Numerical results on $K \rightarrow \pi \nu \bar{\nu}$ and LFVs (μ, τ).

Littlest Higgs model: $SU(5)/SO(5)$ nonlinear sigma model

- $SU(5)$ global symmetry is broken down to $SO(5)$ by a VEV of Σ (15 of $SU(5)$) with a symmetry breaking scale $f \sim \text{TeV}$:

$$\langle \Sigma \rangle = \Sigma_0 = \left(\begin{array}{cc|cc|cc} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ \hline 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ \hline 0 & 1 & 0 & 0 & 0 \end{array} \right).$$

- $SU(5)/SO(5)$: $24 - 10 = 14$ Nambu-Goldstone bosons.

$$\Sigma = \xi \Sigma_0 \xi^T, \quad \xi = \exp(i\Pi/f),$$

$$\Pi = \frac{1}{2} \left(\begin{array}{cc|cc|cc} -\omega^0 - \frac{\eta}{\sqrt{5}} & -\sqrt{2}\omega^+ & -i\sqrt{2}\pi^+ & -i2\phi^{++} & -i\sqrt{2}\phi^+ \\ -\sqrt{2}\omega^- & \omega^0 - \frac{\eta}{\sqrt{5}} & h + i\pi^0 & -i\sqrt{2}\phi^+ & \sqrt{2}(\phi^P - i\phi^0) \\ \hline i\sqrt{2}\pi^- & h - i\pi^0 & \frac{4}{\sqrt{5}}\eta & -i\sqrt{2}\pi^+ & h + i\pi^0 \\ \hline i2\phi^{--} & i\sqrt{2}\phi^- & i\sqrt{2}\pi^- & -\omega^0 - \frac{\eta}{\sqrt{5}} & -\sqrt{2}\omega^- \\ i\sqrt{2}\phi^- & \sqrt{2}(\phi^P + i\phi^0) & h - i\pi^0 & -\sqrt{2}\omega^+ & \omega^0 - \frac{\eta}{\sqrt{5}} \end{array} \right)$$

LHT: gauge symmetries

$$\left(\begin{array}{c|c|c} SU(2)_1 & & \\ \hline & & \\ \hline & & SU(2)_2 \end{array} \right)$$

global:	$SU(5)$ \cup	\xRightarrow{f}	$SO(5)$ \cup
gauged:	$[SU(2) \times U(1)]_1 \times [SU(2) \times U(1)]_2$	\xRightarrow{f}	$SU(2) \times U(1)$ SM electroweak

T-parity: $[SU(2) \times U(1)]_1 \xleftrightarrow{T} [SU(2) \times U(1)]_2 \Rightarrow (g, g')_1 = (g, g')_2$.

Gauge bosons:

$$\left. \begin{array}{l} [W_1^a, B_1] \\ [W_2^a, B_2] \end{array} \right\} \Rightarrow \left\{ \begin{array}{ll} W_L^\pm, Z_L, A_L & \text{“light” SM gauge bosons (T-even)} \\ W_H^\pm, Z_H, A_H & \text{“heavy” } O(f) \text{ masses (T-odd)} \end{array} \right.$$

(pseudo) Nambu-Goldstone bosons:

- $\pi^{\pm,0}, h$ \rightarrow SM Higgs doublet ($\pi^{\pm,0}$ eaten by W_L^\pm, Z_L), T-even.
- $\omega^{\pm,0}, \eta$ \rightarrow eaten by W_H^\pm, Z_H, A_H , T-odd.
- $\phi^{\pm\pm, \pm, 0, P}$ \rightarrow physical, T-odd (effects on flavor obs. suppressed).

LHT: fermion sector

Fermions are doubled for T-parity.

Left-handed

	$SU(2)_1$	$SU(2)_2$	Y_1	Y_2	$SU(2)_L$	Y	T-parity
$q_1^i = \begin{pmatrix} u_1^i \\ d_1^i \end{pmatrix}$	2	1	$\frac{1}{30}$	$\frac{4}{30}$	2	$\frac{1}{6}$	$q_1^i \leftrightarrow -q_2^i$
$q_2^i = \begin{pmatrix} u_2^i \\ d_2^i \end{pmatrix}$	1	2	$\frac{4}{30}$	$\frac{1}{30}$	2	$\frac{1}{6}$	

Right-handed

	$SU(2)_1$	$SU(2)_2$	Y_1	Y_2	$SU(2)_L$	Y	T-parity
u_R^i	1	1	$\frac{1}{3}$	$\frac{1}{3}$	1	$\frac{2}{3}$	+
d_R^i	1	1	$-\frac{1}{6}$	$-\frac{1}{6}$	1	$-\frac{1}{3}$	+
$q_{HR}^i = \begin{pmatrix} u_{HR}^i \\ d_{HR}^i \end{pmatrix}$	nonlinear				2	$\frac{1}{6}$	-

- Singlet top partners T_{\pm} are also introduced to cancel one-loop top Yukawa contribution to SM Higgs boson mass (LH mechanism).
- $u \Rightarrow \nu$ and $d \Rightarrow e$ (with $U(1)$ charge adjustments) for leptons.

$SU(5)$ embedding for Yukawa and gauge interactions

Building blocks of gauge invariant Lagrangian: $SU(5)$ “containers”.

$$\Psi_1(\bar{\mathbf{5}}) = \begin{pmatrix} -i\sigma^2 q_1 \\ 0 \\ 0 \end{pmatrix}, \quad \Psi_2(\mathbf{5}) = \begin{pmatrix} 0 \\ 0 \\ -i\sigma^2 q_2 \end{pmatrix}, \quad \Psi_R = \begin{pmatrix} * \\ * \\ -i\sigma^2 q_{HR} \end{pmatrix}.$$

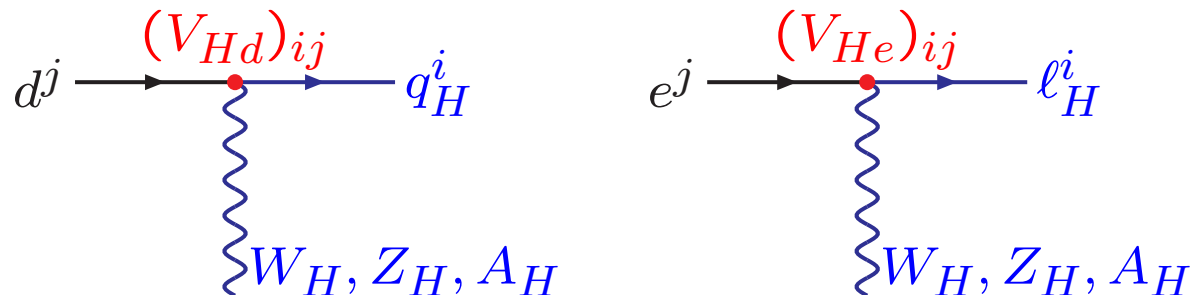
- $\Psi_{1,2}$: linear representations of $SU(5)$. ($\Psi_1 \xleftrightarrow{T} \Psi_2$)
- Ψ_R : $\mathbf{5}$ of $SO(5)$, nonlinear representations of $SU(5)$.
 - ▷ $\Psi'_R = \xi \Psi_R$ transforms as $\mathbf{5}$ of $SU(5)$. ($\xi = \exp(i\Pi/f)$)

New Yukawa coupling $\rightarrow O(f)$ masses of T-odd fermions.

$$\mathcal{L}_H = -\kappa^{ij} f [\bar{\Psi}_2^i \Psi_R'^j - \bar{\Psi}_1^i \tilde{\Psi}_R'^j] + \text{H. c.}, \quad \tilde{\Psi}'_R: \text{T-conjugate of } \Psi'_R.$$

κ^{ij} makes mismatch between light and heavy fermion mass bases.

\Rightarrow new flavor mixings occur in couplings with T-odd gauge bosons.

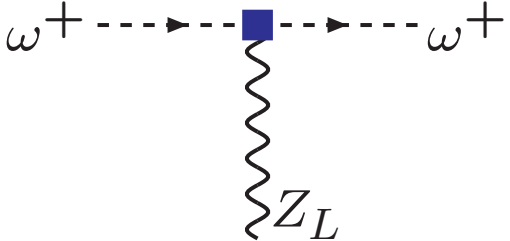


$O(\frac{v^2}{f^2})$ corrections in Z_L coupling

Vacuum configuration after EWSB: $h \rightarrow h + v$ ($v = 246$ GeV).

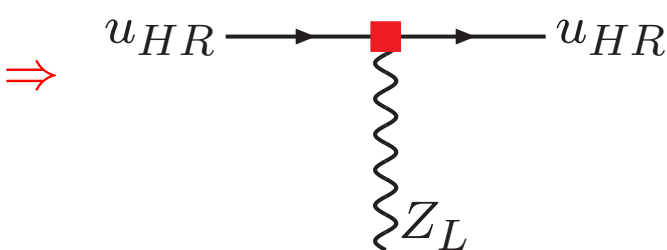
$$\langle \Sigma \rangle_v = \xi_v \Sigma_0 \xi_v^T, \quad \xi_v = 1 + O\left(\frac{v}{f}\right).$$

Expansion around $\langle \Sigma \rangle_v$ leads to $O(\frac{v^2}{f^2})$ corrections in gauge couplings of the pNG bosons and $q_{HR} \Rightarrow$ play central role in Z penguin amplitudes.

$$\mathcal{L} = \frac{f^2}{8} \text{tr} \left[(\mathcal{D}^\mu \Sigma^\dagger) (\mathcal{D}_\mu \Sigma) \right] \Rightarrow \omega^+ \text{---} \text{---} \omega^+ \propto \left(\cos^2 \theta_W - \frac{v^2}{8f^2} \right).$$


[Blanke *et al.*, 2007]

$$\mathcal{L}_{\text{kin}}(q_{HR}) = \frac{1}{2} \left[\bar{\Psi}'_R i \not{D} \Psi'_R + \bar{\tilde{\Psi}}'_R i \not{D} \tilde{\Psi}'_R \right], \quad (\Psi'_R = \xi \Psi_R),$$

$$\Rightarrow u_{HR} \text{---} \text{---} u_{HR} \propto \left(\frac{1}{2} - Q_u \sin^2 \theta_W - \frac{v^2}{8f^2} \right).$$


[Goto *et al.*, 2009; del Águila *et al.*, 2009]

Flavor changing processes in LHT

We studied:

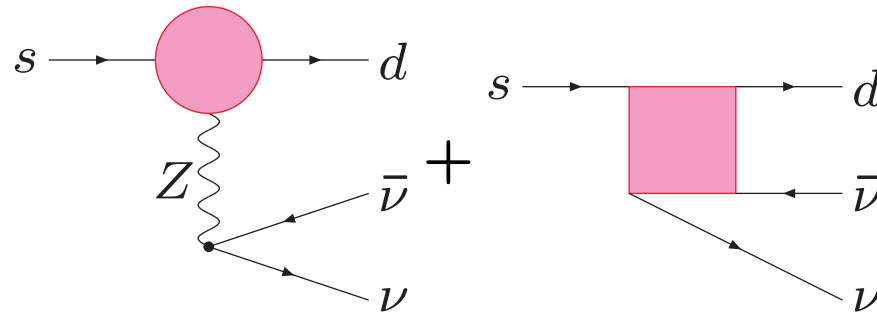
- $K \rightarrow \pi \nu \bar{\nu}$ ($s \rightarrow d$) [PLB670(2009)378],
- LFVs ($\mu \rightarrow e, \tau \rightarrow \mu, \tau \rightarrow e$) [preliminary].

FCNC in LHT: $K \rightarrow \pi \nu \bar{\nu}$ ($s \rightarrow d \nu \bar{\nu}$)

Low energy effective Lagrangian:

$$\mathcal{L}_{[d\nu]}^{\text{eff}} = C_{[d\nu]LL}^{ijklm} (\bar{d}^i \gamma^\mu d_L^j) (\bar{\nu}^l \gamma_\mu \nu_L^m), \quad i \neq j.$$

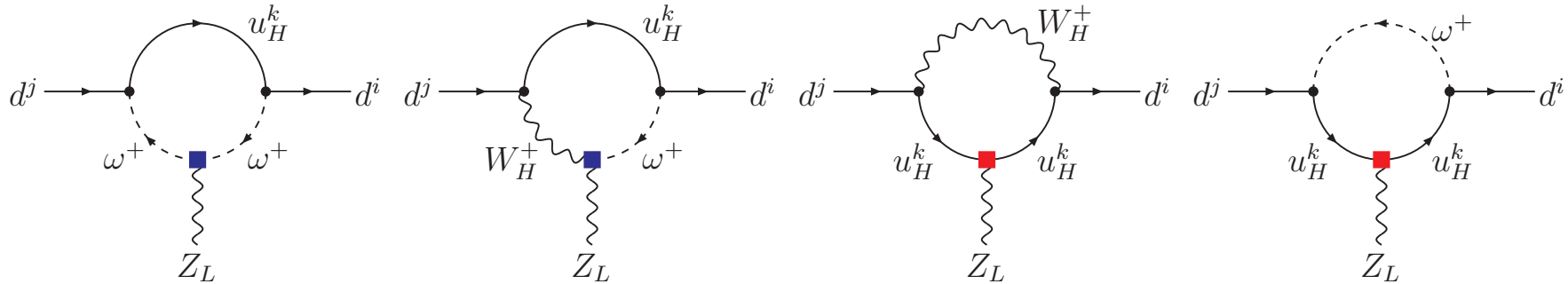
- Right-handed $d^j \rightarrow d^i$ current is suppressed in LHT (as in SM).
- Leading order = one-loop (no tree level contributions).



- In 't Hooft–Feynman gauge, box terms are manifestly finite.
- T-even sector = SM \oplus $SU(2)$ singlet vector-like top partner T_+ .
 - ▷ $C_{[d\nu]LL}^{ijkl}(\text{T-even}) \propto \lambda_t^{(ij)} = (V_{\text{CKM}}^*)_{ti} (V_{\text{CKM}})_{tj}$.

T-odd loops: Z penguins

- Zeroth order terms in $\frac{v^2}{f^2}$ expansion vanishes.
- Relevant diagrams for leading $O(\frac{v^2}{f^2})$ contributions:



- UV divergences cancel in the sum \Rightarrow finite amplitude obtained.

$$C_{[d\nu]LL}^{ijklm}(\text{T-odd}) = -\frac{g^4}{(4\pi)^2 m_{W_L}^2} \sum_{k,n=1}^3 \lambda_k^{Hd(ij)} \lambda_n^{H\nu(lm)} \times \frac{v^2}{16f^2} X(z_k, y_n, r), \quad i \neq j,$$

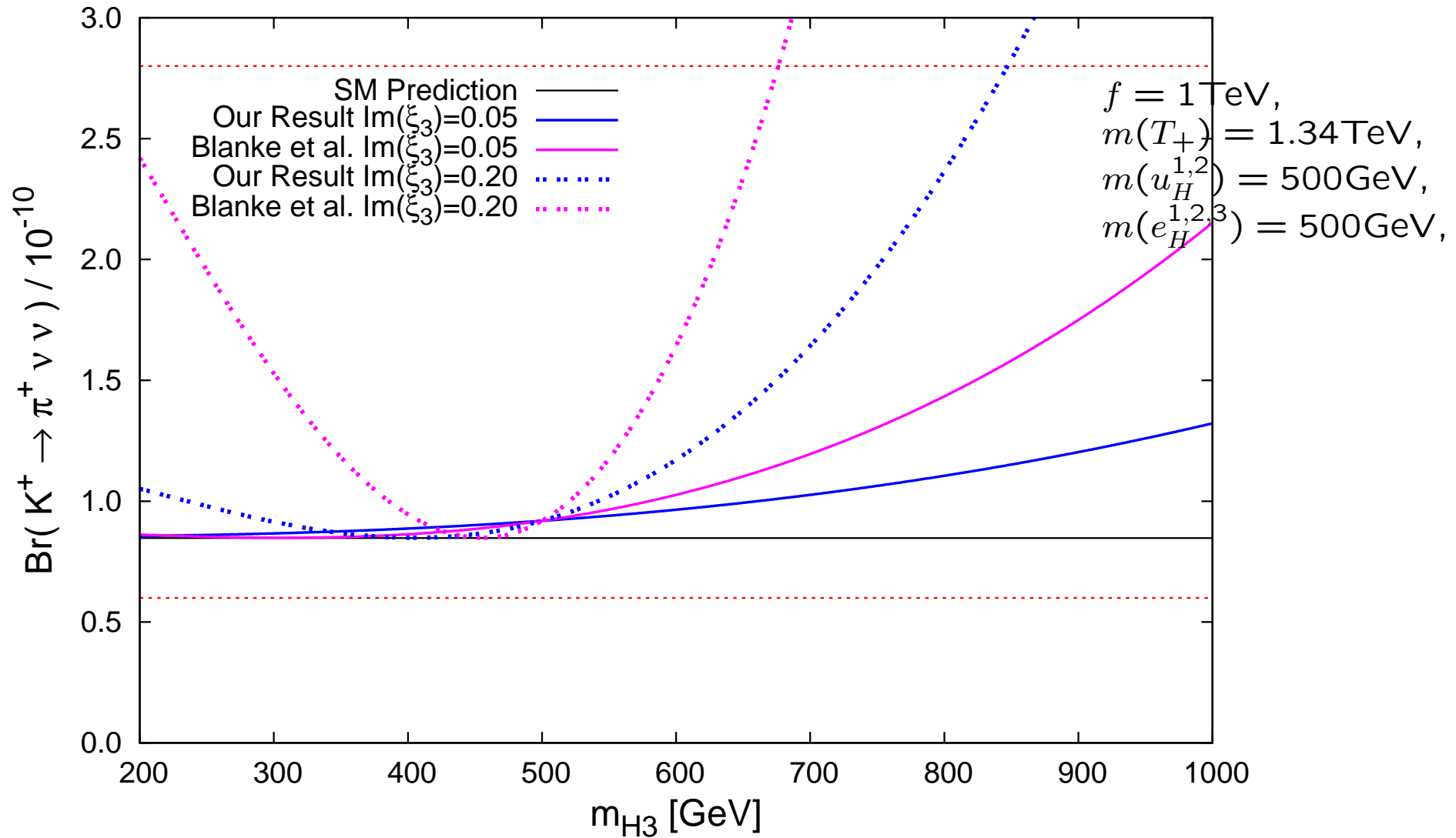
$$\lambda_k^{Hd(ij)} = (V_{Hd}^*)_{ki} (V_{Hd})_{kj}, \quad \lambda_n^{H\nu(lm)} = (V_{H\nu}^*)_{nl} (V_{H\nu})_{nm},$$

$$z_k = m_{q_H^k}^2 / m_{W_H}^2, \quad y_n = m_{\ell_H^n}^2 / m_{W_H}^2, \quad r = m_{A_H}^2 / m_{Z_H}^2.$$

X : Inami-Lim type function (Z-penguin + box).

$$B(K^+ \rightarrow \pi^+ \nu \bar{\nu})$$

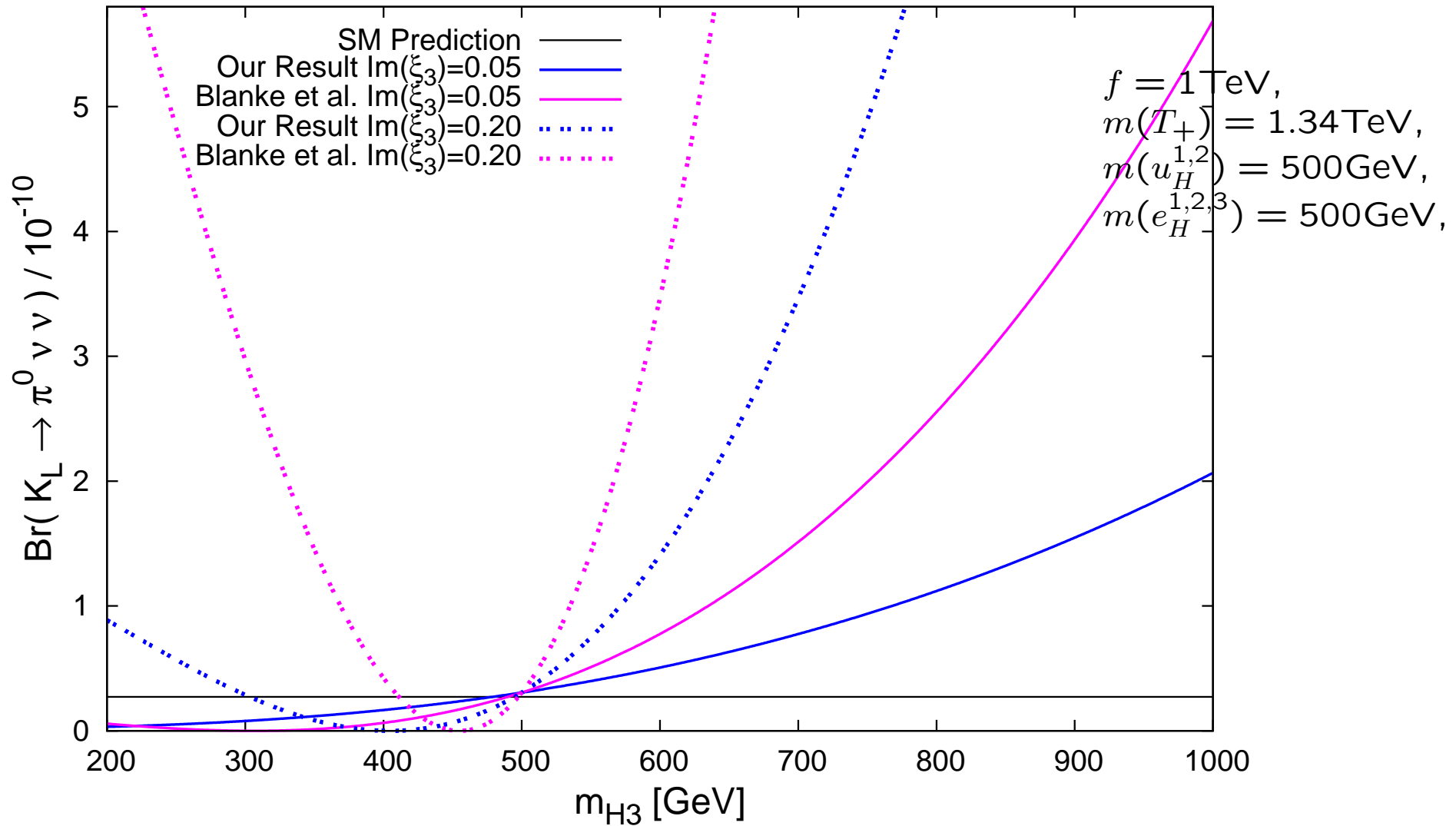
$$B(\text{exp}) = (1.5_{-0.9}^{+1.3}) \times 10^{-10}$$



$$\xi_3 = (V_{Hd})_{31}(V_{Hd})_{32}^*. \quad \text{Re}[(V_{Hd})_{31}^*(V_{Hd})_{32}] = 0 \quad (\text{to suppress T-odd contrib. to } \varepsilon_K).$$

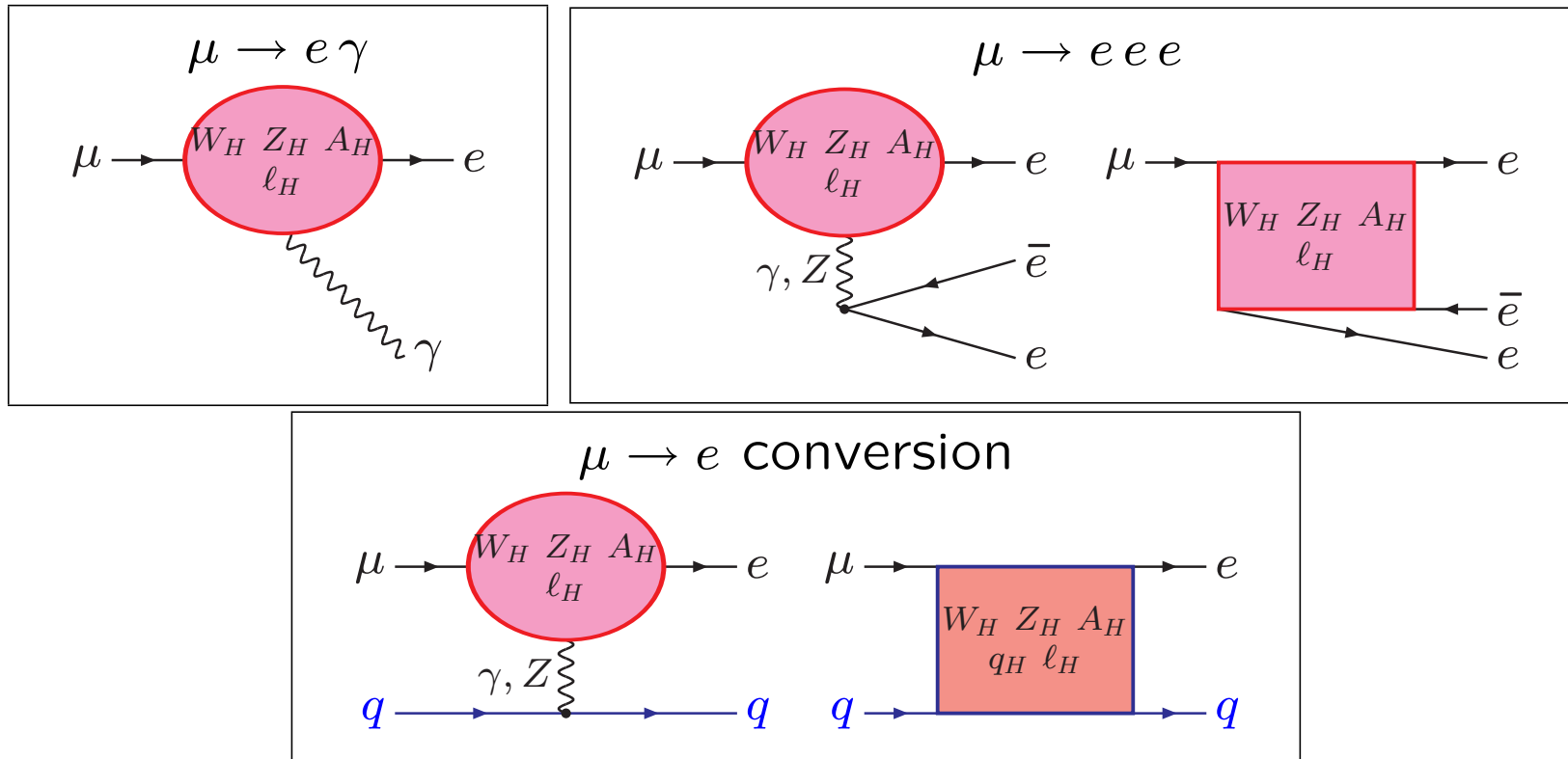
$$B(K_L \rightarrow \pi^0 \nu \bar{\nu})$$

$$B(\text{exp}) < 2.1 \times 10^{-7}$$



$$\xi_3 = (V_{Hd})_{31}(V_{Hd})_{32}^*. \quad \text{Re}[(V_{Hd})_{31}^*(V_{Hd})_{32}] = 0 \quad (\text{to suppress T-odd contrib. to } \varepsilon_K).$$

μ LFV: $\mu \rightarrow e \gamma$, $\mu \rightarrow e e e$ and $\mu \rightarrow e$ conversion

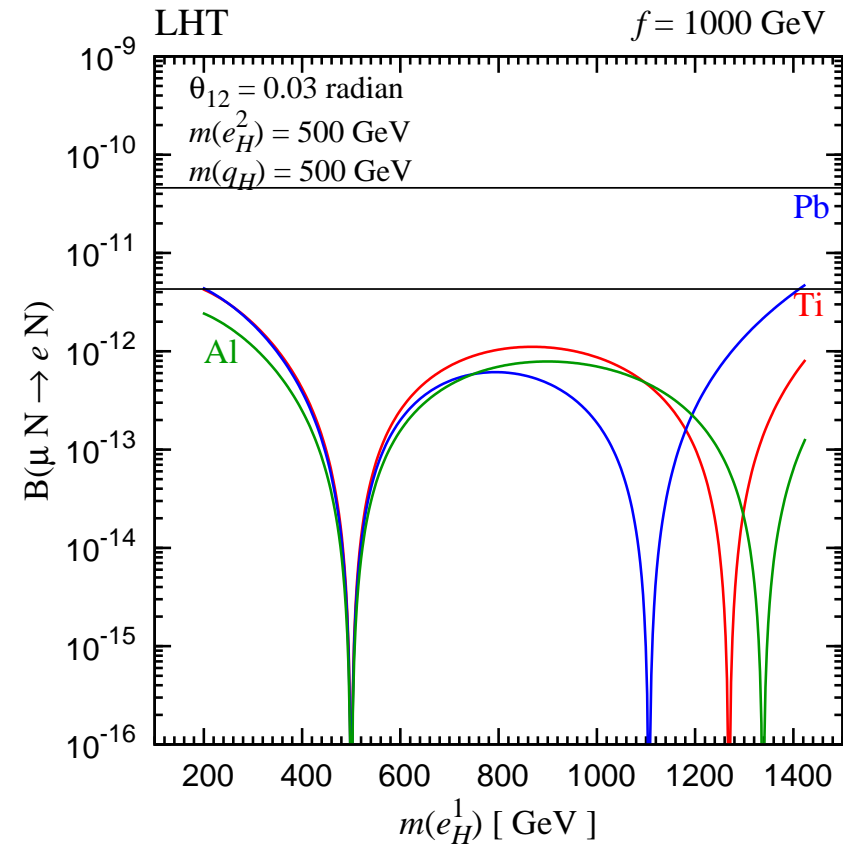
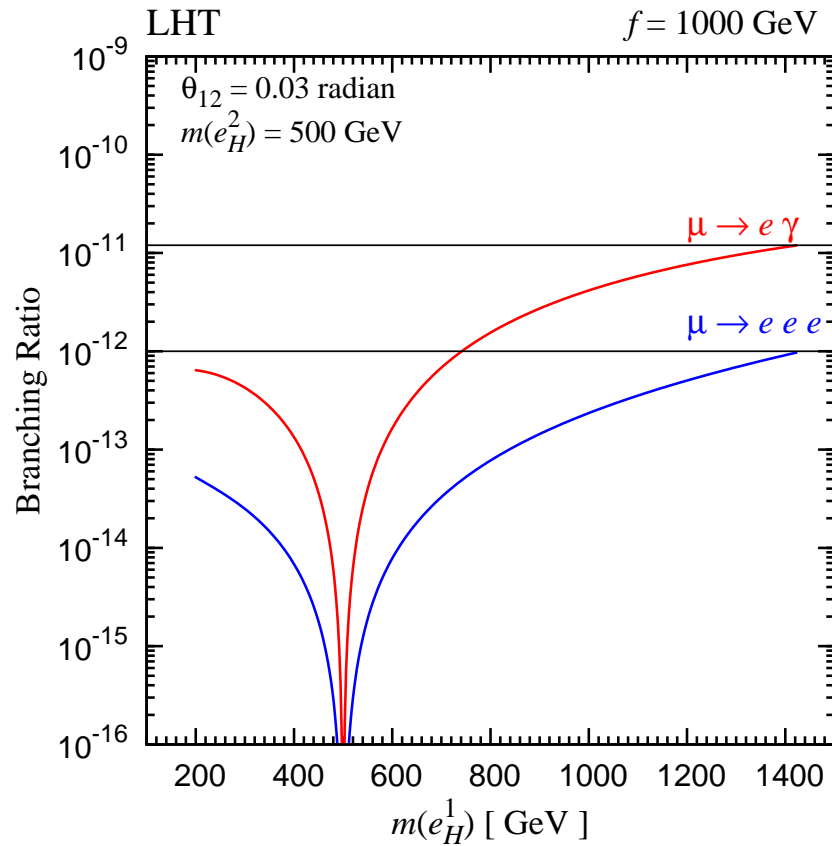


- T-odd loops only.
- Calculated in the same way as quark FCNC: $(d, s, b) \Rightarrow (e, \mu, \tau)$.

See also: del Águila, Illana & Jenkins, JHEP0901(2009)080;

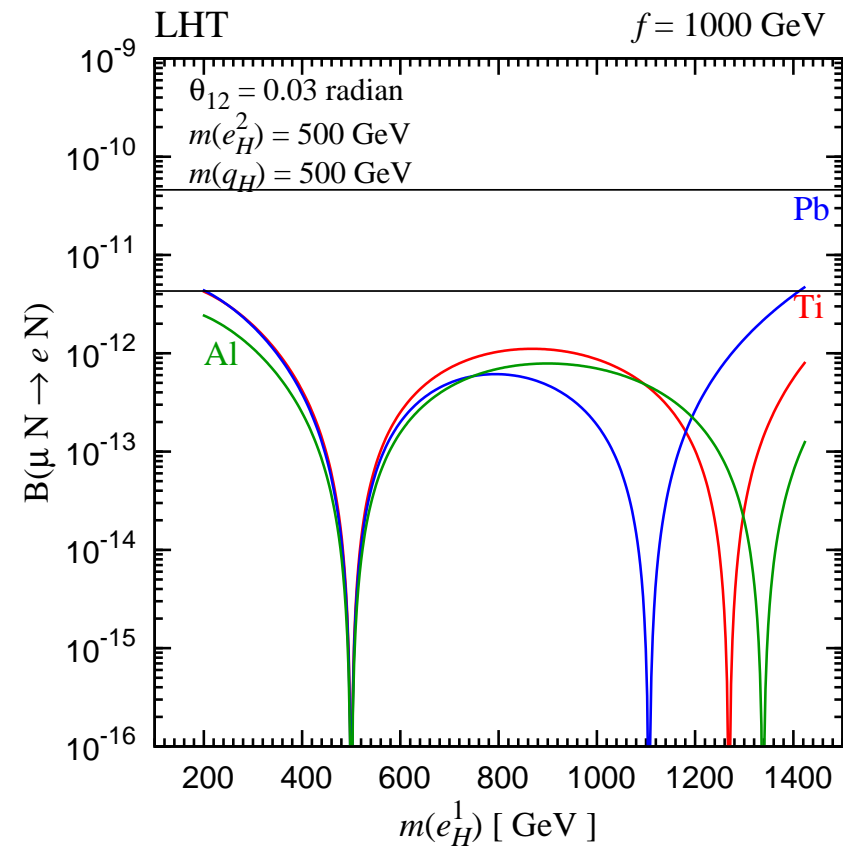
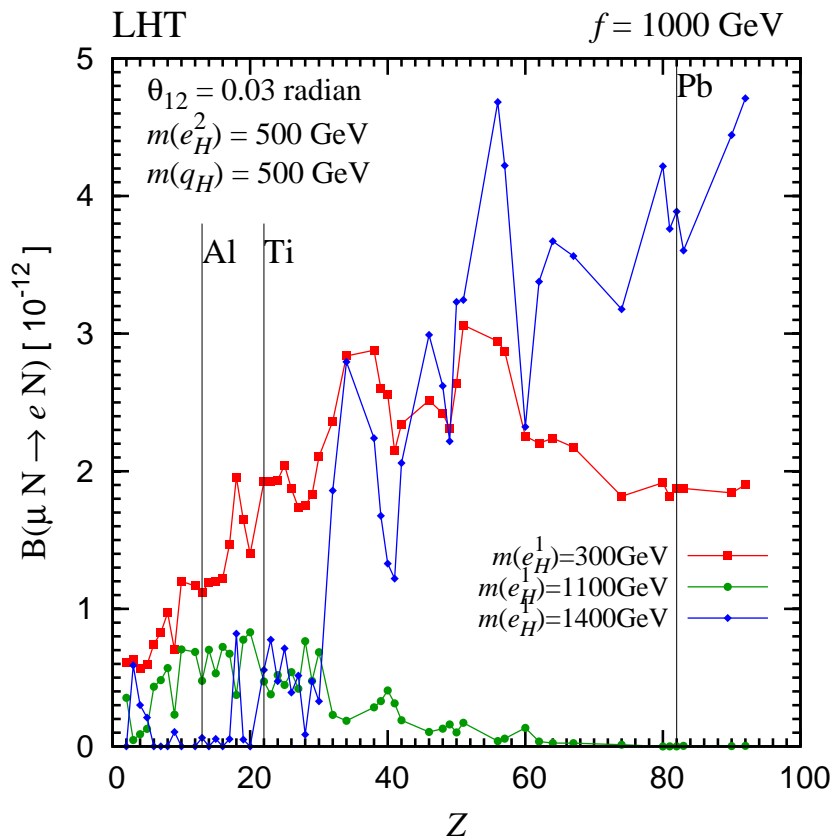
Blanke *et al.*, JHEP05(2007)013; arXiv:0906.5454.

μ LFBV: $\mu \rightarrow e \gamma$, $\mu \rightarrow e e e$ and $\mu \rightarrow e$ conversion



- Assumptions: pure 1-2 mixing ($\theta_{i3} = 0$ in V_{He} , $m(e_H^2) = m(e_H^3)$); q_H degenerate in mass.
- Nuclear matrix elements for $\mu \rightarrow e$ conv. available in Kitano *et al.*, PRD66(2002)096002.

μ LFBV: $\mu \rightarrow e$ conversion: atomic number dependence



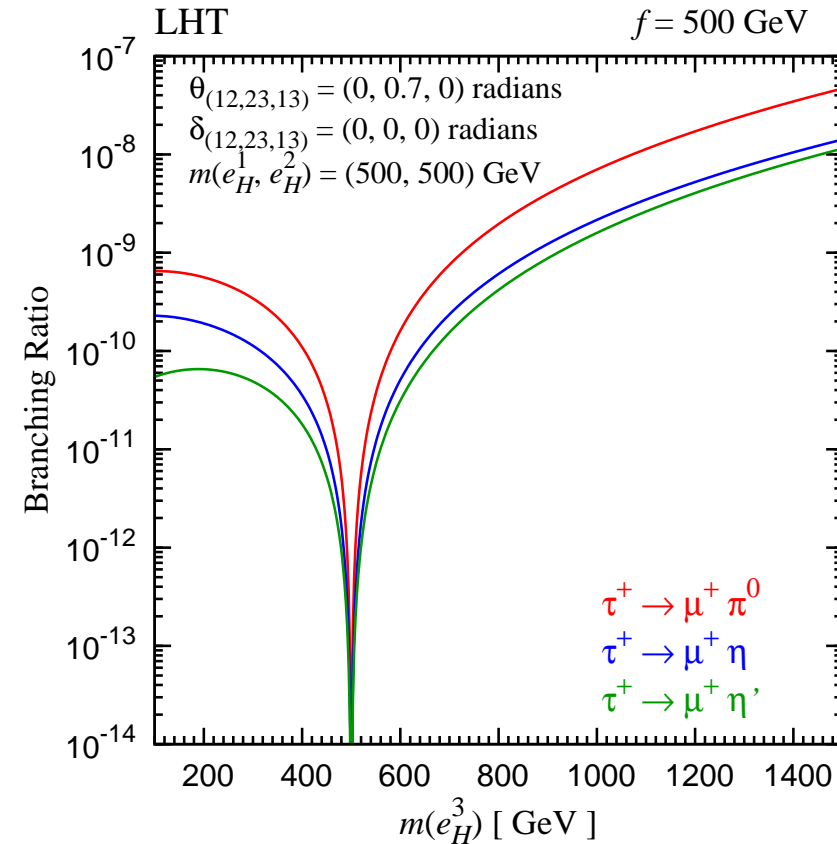
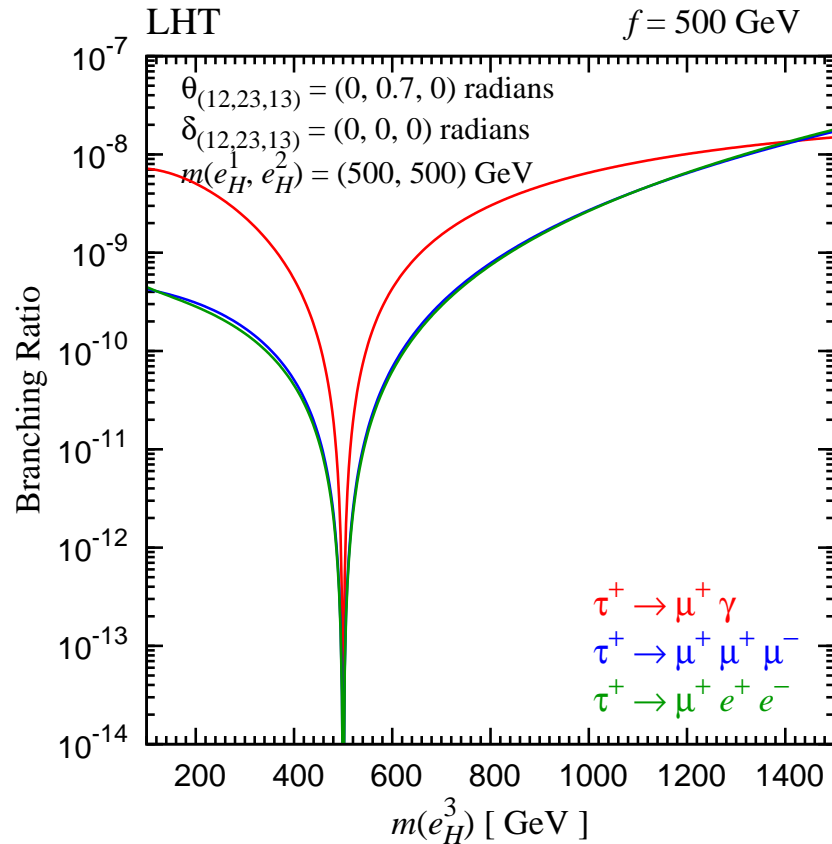
- B =conversion rate/capture rate depends on nucleus.
 - ▷ Cancellation among amplitudes possible.
 - ▷ Measurements with various Z useful.
- Experiments aiming 10^{-16} planned (COMET@J-PARC, Mu2e@FNAL).

τ LFV

	$\mu \rightarrow e$ relatives
$\tau \rightarrow \mu \gamma$ $\tau \rightarrow e \gamma$	$\mu \rightarrow e \gamma$
$\tau \rightarrow \mu \mu \mu$ $\tau \rightarrow e e e$	$\mu \rightarrow e e e$
$\tau \rightarrow \mu e^+ e^-$ $\tau \rightarrow e \mu^+ \mu^-$	
$\tau \rightarrow \mu + \text{hadrons}$ $\tau \rightarrow e + \text{hadrons}$	$\mu N \rightarrow e N$

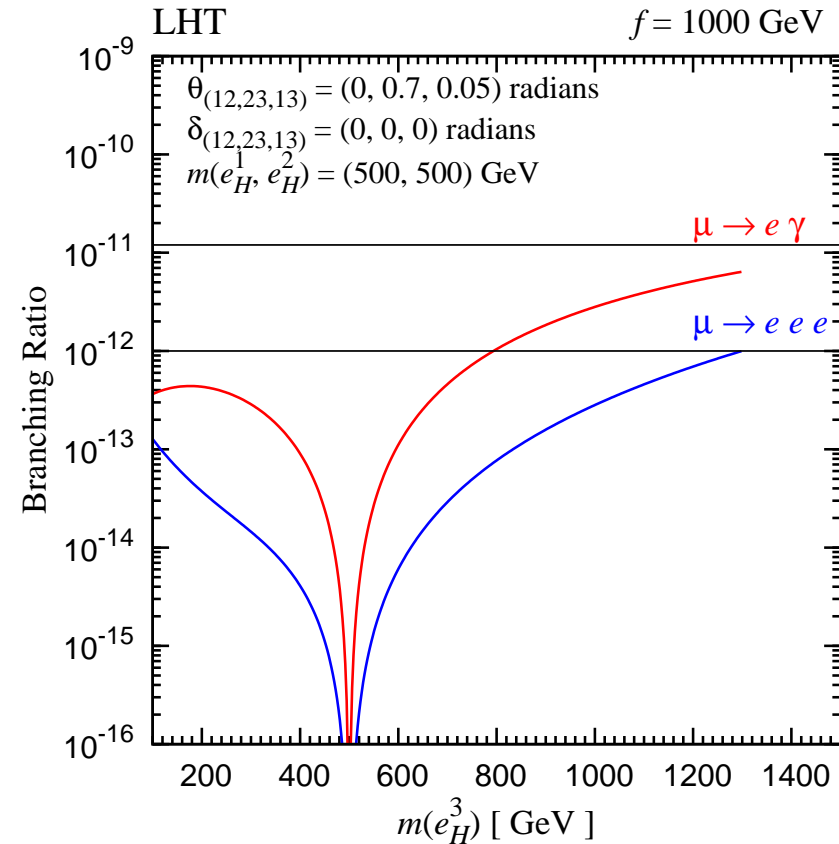
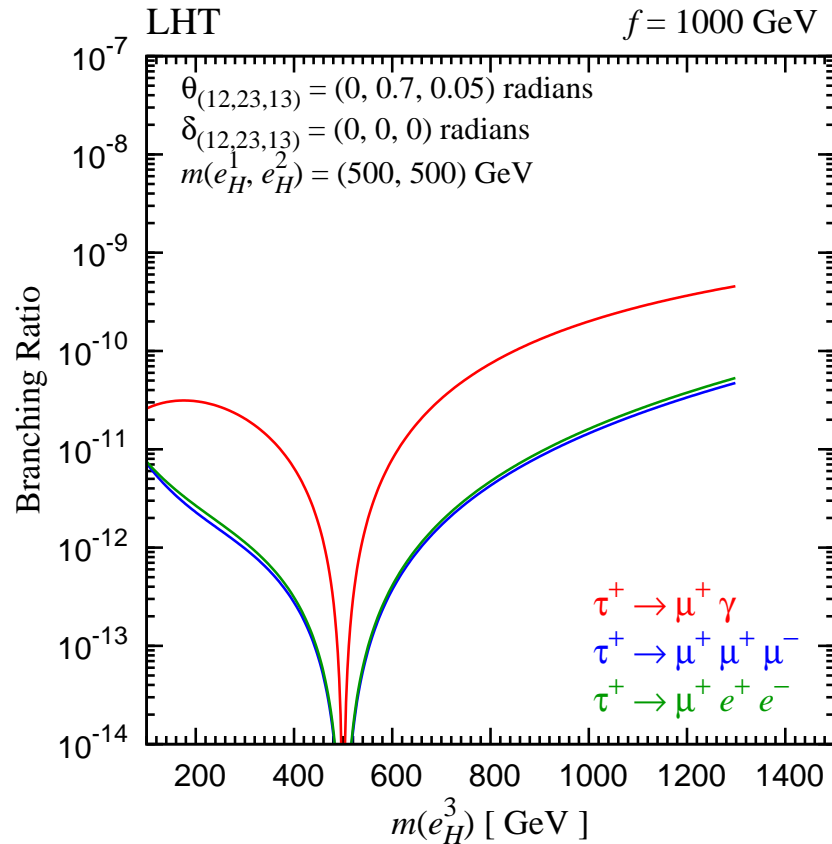
- $B(\text{exp}) \lesssim 10^{-8} - 10^{-7}$ (B-factories).
- Processes such as $\tau^+ \rightarrow \mu^- e^+ e^+$ ($\Delta LF > 1$) or $\tau \rightarrow \mu K^0$ (LFV and QFV) are not shown here.

τ LNV



- Assumptions: pure 2-3 mixing ($\theta_{1j} = 0$ in V_{He} , $m(e_H^1) = m(e_H^2)$); q_H degenerate in mass.
 - ▷ $\mu \rightarrow e$ and $\tau \rightarrow e$ are suppressed.

τ LFV and μ LFV



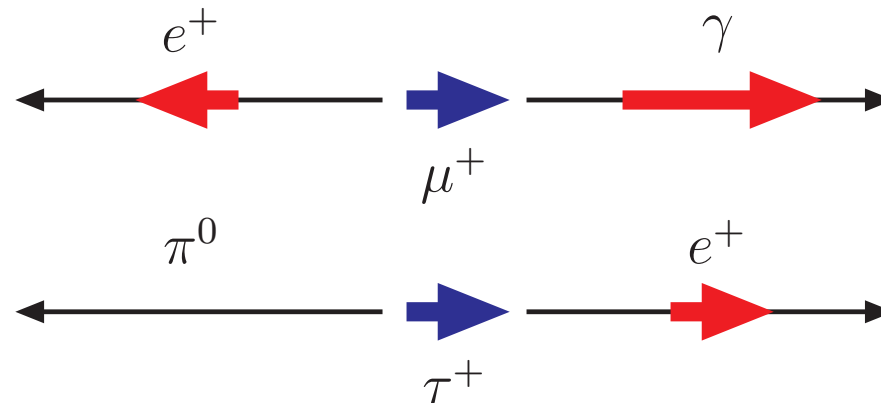
- 2-3 ($\tau - \mu$) \otimes 1-3 ($\tau - e$) mixings $\Rightarrow \mu - e$ generated.
- Constraints from $\mu \rightarrow e$ transitions strong for general V_{He} and $m(e_H^i)$.

Comments on LFV

- T-odd gauge bosons couple to **left-handed** T-even leptons/quarks.
- ⇒ Chirality structure of effective $\ell \rightarrow \ell'$ interactions are restricted to:

$$(\bar{e}_L \gamma^\mu \mu_L) (\bar{f} \gamma_\mu (g_v - g_a \gamma_5) f), \quad m_\mu (\bar{e}_L \sigma^{\mu\nu} \mu_R) F_{\mu\nu}, \quad m_e (\bar{e}_R \sigma^{\mu\nu} \mu_L) F_{\mu\nu}.$$

- ▷ Four-fermi effective interactions dominate over $\sigma^{\mu\nu} F_{\mu\nu}$ terms in trilepton/semihadronic modes. $B(\mu \rightarrow eee) / B(\mu \rightarrow e\gamma) > O(\alpha)$. (cf. $\tan \beta$ enhancement in SUSY)
- ▷ Polarization asymmetries may be significant.



* More P/CP odd polarization asymmetries in trilepton modes.

Conclusion

- FCNC and LFV processes in the Littlest Higgs model with T-parity are (re-)studied.
- One-loop $d^i \rightarrow d^j$ and $e^i \rightarrow e^j$ amplitudes are UV finite at $O(\frac{v^2}{f^2})$.
- Branching fractions of $K \rightarrow \pi \nu \bar{\nu}$ decays may differ from SM predictions significantly.
- Lepton flavor violations may be within the reach of near future experiments.
 - ▷ Various patterns of signals (different from SUSY cases) possible.

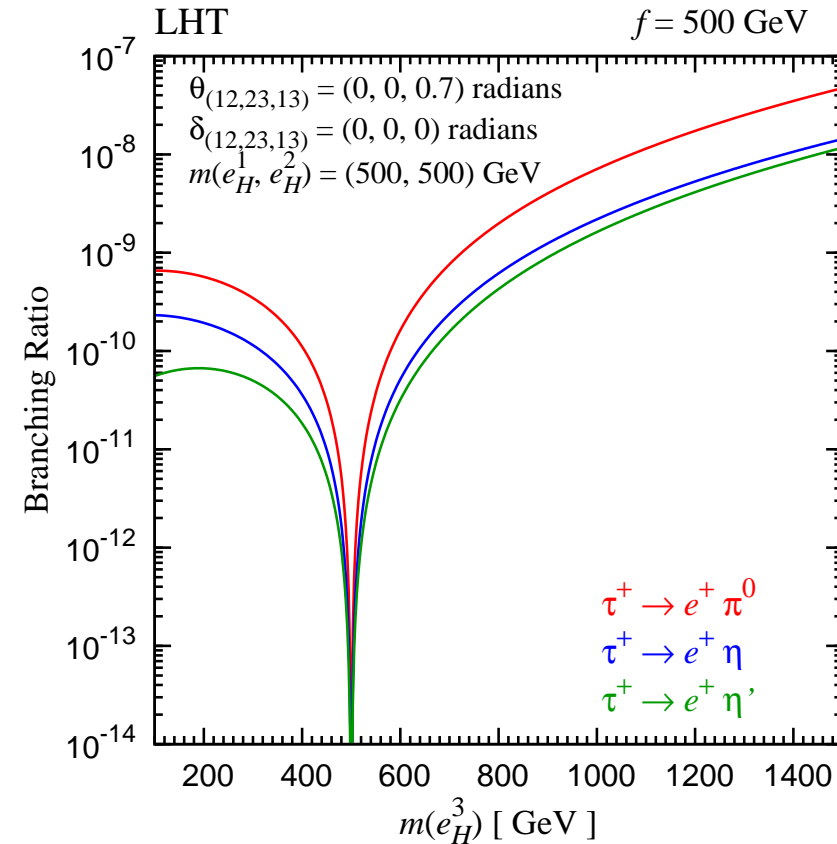
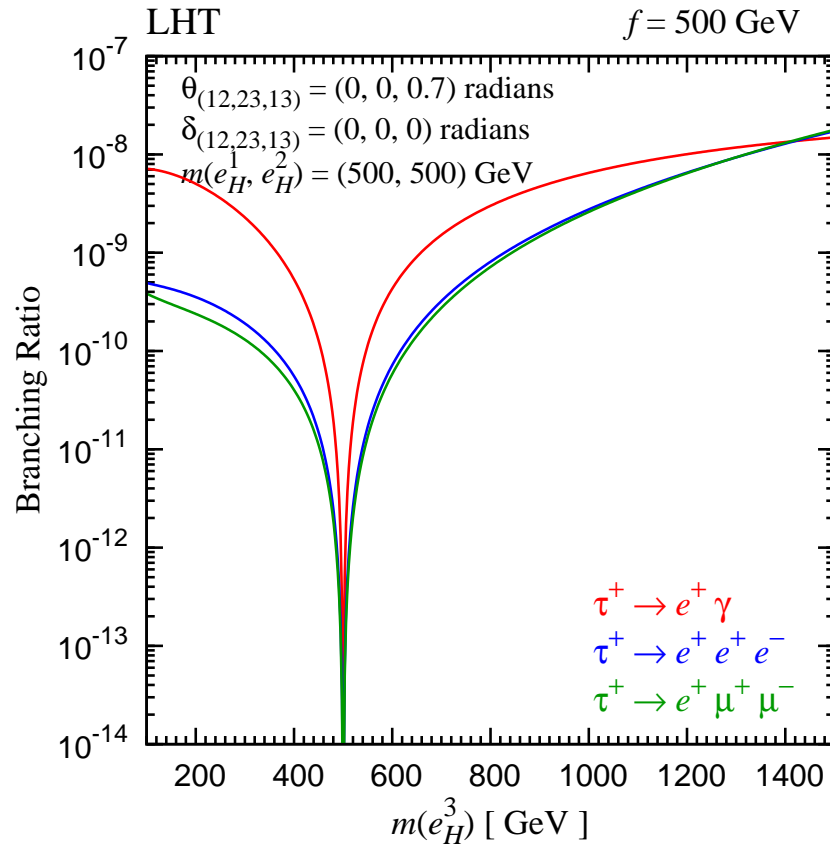
Backups

Input parameters

There are 20 input parameters relevant for flavor observables:

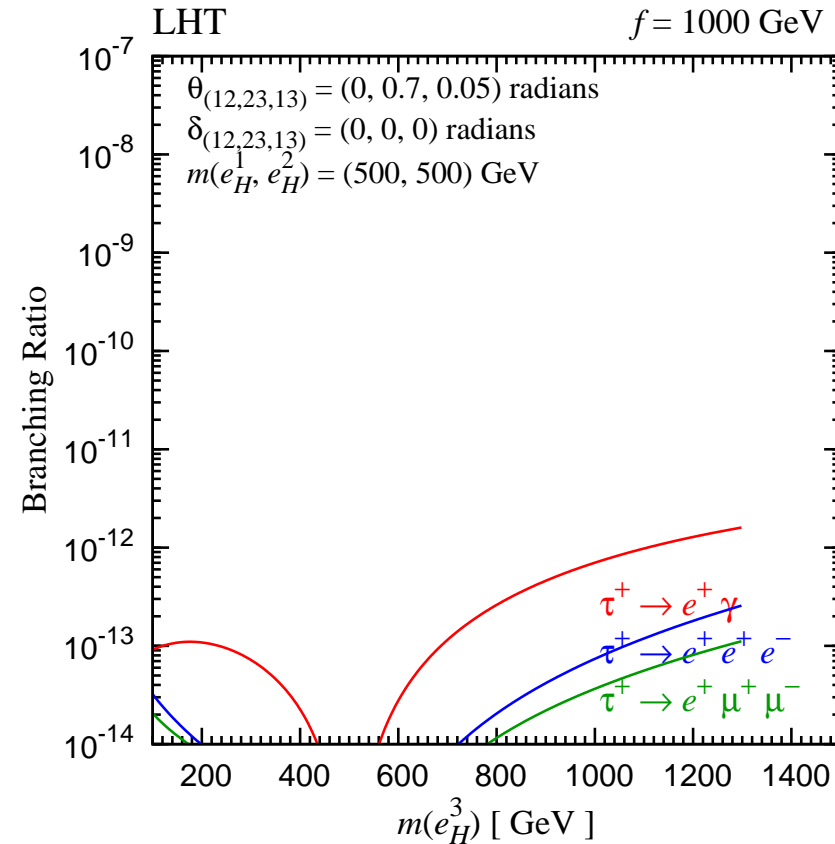
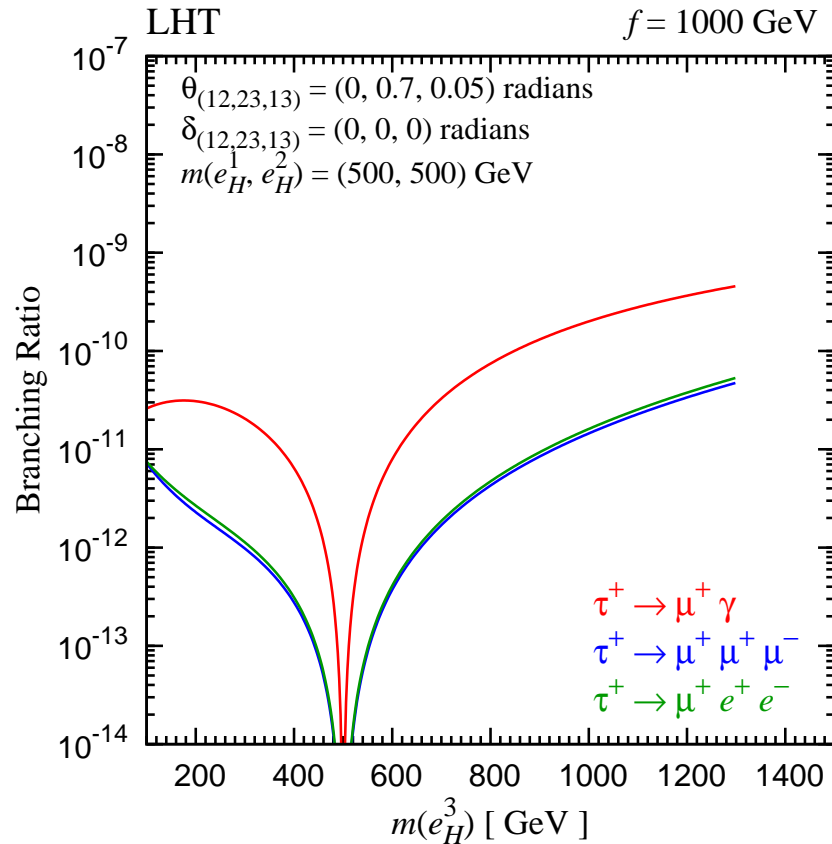
- $SU(5) \rightarrow SO(5)$ SSB scale f .
- $t - T_+$ mixing parameter x_L .
 - ▷ EW precision constraints $\Rightarrow f \gtrsim 500\text{GeV}$ and $x_L \sim 0.5$.
- 6 masses of T-odd fermions $m(q_H^i)$ and $m(e_H^i)$.
- 6 mixing angles and 6 phases in mixing matrices V_{Hd} and V_{He} .

τ LNV



- Assumptions: pure 1-3 mixing ($\theta_{2j} = 0$ in V_{He} , $m(e_H^1) = m(e_H^2)$); q_H degenerate in mass.
 - ▷ $\mu \rightarrow e$ and $\tau \rightarrow \mu$ are suppressed.

τ LFV and μ LFV



- 2-3 ($\tau - \mu$) \otimes 1-3 ($\tau - e$) mixings $\Rightarrow \mu - e$ generated.
- Constraints from $\mu \rightarrow e$ transitions strong for general V_{He} and $m(e_H^i)$.