Constraints on the Timeon Model

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What is the "timeon" ?

Theory of Timeon [Friedberg and Lee, $arXiv:0809.3633$]

Friedberg and Lee proposed a new spontaneous CP violation mechanism. They introduced a new gauge singlet pseudo-scalar $\tau(x)$ and new Yukawa type interactions, $\bar{q}[i\gamma_5\tau(x)F]q$

$$\mathcal{H} = \bar{u}_i[M_u + i\gamma_5\tau_0 F]_{ij}u_j + \bar{d}_i[M_d + i\gamma_5\tau_0 F]_{ij}d_j .$$

$$\phi(x)Y_{ij} \to vY_{ij} = M_{ij}$$

F is a 3×3 real matrix and τ_0 is the VEV of a new gauge singlet pseudo-scalar, $\tau(x)$.

$$\bar{q}[i\gamma_5\tau(x)F]q \to \bar{q}[i\gamma_5\tau_0 F]q$$

They assumed M_q and F are real, and $\det M_q = 0$.

- CP and T symmetries are spontaneously broken after the pseudo-scalar (timeon) gets the VEV,
- the pseudo-scalar is also responsible for the up and down quark masses.

What is the "timeon" ?

Theory of Timeon [Friedberg and Lee, $arXiv$: 0809.3633]

Friedberg and Lee proposed the following quark mass matrices

$$\mathcal{H} = \bar{u}_L[M_u + i\tau_0 F]u_R + \bar{d}_L[M_d + i\tau_0 F]d_R + h.c.$$

$$M_q = \begin{pmatrix} b_q\eta_q^2(1+\xi_q^2) & -b_q\eta_q & -b_q\xi_q\eta_q \\ -b_q\eta_q & b_q + a_q\xi_q^2 & -a_q\xi_q \\ -b_q\xi_q\eta_q & -a_q\xi_q & a_q + b_q \end{pmatrix} \quad q = u, d$$

where $a_q,\ b_q,\ \eta_q,\ \xi_q$ are assumed to be real, and the mass matrix is invariant under the translational family symmetry:

FL symmetry

$$q_1 \to q_1 + z, \quad q_2 \to q_2 + \eta\ z, \quad q_3 \to q_3 + \eta\xi\ z.$$

F is also 3×3 matrix described by two angles and given by

$$F = \begin{pmatrix} \cos^2\alpha & \sin\alpha\cos\alpha\cos\beta & \sin\alpha\cos\alpha\sin\beta \\ \sin\alpha\cos\alpha\cos\beta & \sin^2\alpha\cos^2\beta & \sin^2\alpha\sin\beta\cos\beta \\ \sin\alpha\cos\alpha\sin\beta & \sin^2\alpha\sin\beta\cos\beta & \sin^2\alpha\sin^2\beta \end{pmatrix}.$$

Since $\det M_q = 0$, m_u and m_d may be proportional to the VEV, τ_0 .

τ_0 may be much smaller that the EW scale

What is the "timeon" ?

Theory of Timeon [Friedberg and Lee, $arXiv$: 0809.3633]

Let us estimate the magnitude of the timeon VEV, τ_0 .

- CKM matrix

  M_q can be diagonalized by the following matrix:

$(q = u, d)$

$$U_q = \begin{pmatrix} \cos\theta_q & -\sin\theta_q & 0 \\ \sin\theta_q\cos\phi_q & \cos\theta_q\cos\phi_q & -\sin\phi_q \\ \sin\theta_q\sin\phi_q & \cos\theta_q\sin\phi_q & \cos\phi_q \end{pmatrix} .$$

$$\eta_q = \tan\theta_q\cos\phi_q$$
$$\xi_q = \tan\phi_q$$

[Ansatz]

$\theta_u = \theta_c \simeq 13.1^\circ$
$\phi_u = \pi + \epsilon$

$\theta_d = 0$
$\phi_d = \pi - \gamma$

$$U_u \simeq \begin{pmatrix} \cos\theta_c & -\sin\theta_c & 0 \\ -\sin\theta_c & -\cos\theta_c & \epsilon \\ -\epsilon\sin\theta_c & -\epsilon\cos\theta_c & -1 \end{pmatrix} , \quad U_d = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -\cos\gamma & -\sin\gamma \\ 0 & \sin\gamma & -\cos\gamma \end{pmatrix} .$$

$$V_{CKM} = U_u^\dagger U_d + \delta U(\tau_0)$$

We ignore $\delta U(\tau_0)$ term here, then ϵ and γ can be determined.

$\epsilon \simeq 1.03^\circ$
$\gamma \simeq 1.26^\circ$

$\eta_u \simeq -0.233 \quad \xi_u \simeq 0.018$
$\eta_d = 0 \quad \xi_d \simeq -0.022$

What is the “timeon” ?

Theory of Timeon [Friedberg and Lee, $arXiv$: 0809.3633]

Let us estimate the magnitude of the timeon VEV, τ_0 .

- up- and down-quark masses

  M_q can be diagonalized by U_q and leads to the following eigenvalues

$$\lambda_q(1) = 0, \quad \lambda_q(2) = b_q[1 + \eta_q^2(1 + \xi_q^2)], \quad \lambda_q(3) = a_q(1 + \xi_q^2) + b_q.$$

  Quark masses are given by $m_q(i) = \lambda_q(i) + \delta\Lambda_q^i(\tau_0)$. Here we assume

$$m_q(2) = \lambda_q(2), \quad m_q(3) = \lambda_q(3).$$

  On the other hand

$$\det[(M_q + i\tau_0 F)(M_q + i\tau_0 F)^\dagger] = m_q^2(1)\ m_q^2(2)\ m_q^2(3).$$

  Therefore the light quark masses are given as

$$m_u(1) = \tau_0 \sin^2\alpha \cos^2(\beta + \theta_c) + \mathcal{O}(\tau_0^2),$$

$$m_d(1) = \tau_0 \sin^2\alpha \cos^2\beta + \mathcal{O}(\tau_0^2).$$

  Given $m_u(1)/m_d(1) \simeq 0.5$ and $\theta_c \simeq 13.1^\circ$

$$\beta \simeq 48^\circ.$$

What is the "timeon" ?

Theory of Timeon [Friedberg and Lee, $arXiv$: 0809.3633]

Let us estimate the magnitude of the timeon VEV, τ_0.

- Jarlskog invariant [Jarlskog, PRD35 (1978)]

$$\mathcal{J} = Im[V_{11}V_{22}V_{12}^*V_{21}^*]$$

$$\simeq \frac{m_d(1)}{\sin^2\alpha\cos^2\beta}\left[\frac{\sin\alpha\cos(\beta+\theta_c)}{m_d(2)}A + \frac{\cos\alpha\sin\gamma - \sin\alpha\sin\beta\cos\gamma}{m_d(3)}B + \frac{-\cos\alpha\cos\gamma - \sin\alpha\sin\beta\sin\gamma}{m_d(2)+m_d(3)}C\right]$$

$A \simeq -2\times 10^{-4},\ B \simeq 8.8\times 10^{-3},\ C \simeq 1.1\times 10^{-3},$

$m_d(2) \simeq 95\ \text{MeV},\ m_d(b) \simeq 1.25\ \text{GeV},$

$\beta \simeq 48^\circ,\ \gamma \simeq 1.26^\circ,\ \theta_c \simeq 13.1^\circ,$

$\mathcal{J}^{exp} \simeq 3.08\times 10^{-5}.$

$\alpha \simeq -36^\circ$ ⇒ $\tau_0 \simeq 33\ \text{MeV}$

$(\ m_d(1) \simeq \tau_0 \sin^2\alpha\cos^2\beta\ )$

What is the "timeon" ?

Theory of Timeon [Friedberg and Lee, $arXiv$: 0809.3633]

The potential of the timeon field is given by

$$V(\tau) = -\frac{1}{2}\lambda\tau^2\left(\tau_0^2 - \frac{1}{2}\tau^2\right).$$

Expanding $V(\tau)$ around $\tau = \tau_0$, we have

$$V(\tau) \sim -\frac{1}{4}\lambda\tau_0^4 + \underline{\frac{1}{2}M_T(\tau-\tau_0)^2} + \cdots ,$$

where $M_T = \sqrt{2\lambda}\,\tau_0$ is the mass of a new quantum, timeon.

Since the timeon is responsible for the light quark masses, its VEV cannot be much larger than electroweak scale.

$$m_d(1) \simeq \tau_0 \sin^2\alpha\cos^2\beta = 3.5 \sim 6.0 \text{ MeV}$$

For instance they estimated that $\tau_0 \simeq 33$ MeV , which implies the lower bound of the timeon mass

$$\boxed{M_T < 47 \text{ MeV} \quad (\text{for } \lambda < 1).}$$

FCNCs

Problem

A light timeon could cause dangerous FCNC processes!!

In general, M_q and F cannot be diagonalized simultaneously.

$$\bar{q}_L[M_q + i\tau_0 F]q_R \to \bar{q}_L^m\ U_q^\dagger[M_q + i\tau_0 F]U_q^*\ q_R^m$$
$$\equiv \bar{q}_L^m[M_q^{'} + i\tau_0 F^q]q_R^m$$
$$= \mathrm{diag}(\ m_q(1),\ m_q(2),\ m_q(3)\ )$$

Each $M_q^{'}$ and F^q are not diagonal in general.
In fact, the off diagonal elements can be calculated as

$$F^u \equiv U_u^T F U_u = \begin{pmatrix} 0.08 & -0.14 & 0.23 \\ -0.14 & 0.25 & -0.41 \\ 0.23 & -0.41 & 0.67 \end{pmatrix}$$

$$F^d \equiv U_d^T F U_d = \begin{pmatrix} 0.15 & -0.18 & 0.31 \\ -0.18 & 0.21 & -0.36 \\ 0.31 & -0.36 & 0.64 \end{pmatrix}$$

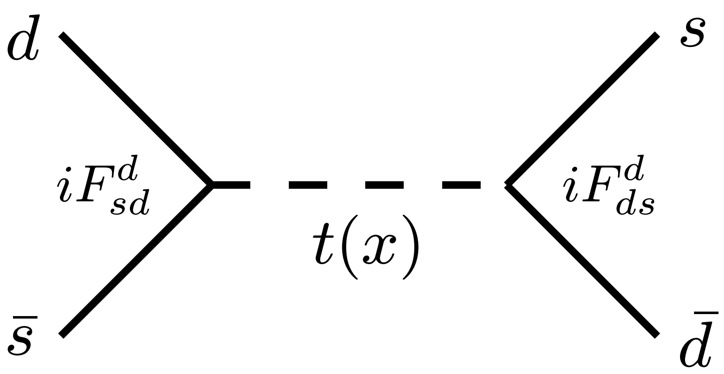
Note
$\tau_0 = 33$ MeV corresponds to
$\eta_u \simeq -0.233,\ \xi_u \simeq 0.018,$
$a_d \simeq 173.76,\ b_u \simeq 1.186,$
$\eta_d = 0,\ \xi_u \simeq -0.022,$
$a_d \simeq 4.103,\ b_d \simeq 0.095,$
$\alpha \simeq -36^\circ,\ \beta \simeq 48^\circ.$

FCNCs

Thoes off diagonal elements of F^q cause flavor changing timeon interactions such as

$$iF^u_{ij}\bar{u}^m_{Li}\ t\ u^m_{Rj},\quad iF^d_{ij}\bar{d}^m_{Li}\ t\ d^m_{Rj}. \quad (i \neq j)$$

For example, $K^0 - \overline{K^0}$ mixing process occurs **at tree level**.



However, FCNC processes are strongly constrained by experiments.

FCNCs

For instance, the contribution form the timeon to the mass mixing parameter in the neutral K meson system can be estimated as

$$\Delta M_K^{timeon} = 2 \left| \frac{2F_{ds}^d (F_{sd}^d)^*}{M_T^2} < K^0 | \bar{d}_L^\alpha s_R^\alpha \bar{d}_R^\beta s_L^\beta | \bar{K}^0 > + \frac{(F_{ds}^d)^2 + (F_{sd}^d)^{*2}}{M_T^2} < K^0 | \bar{d}_L^\alpha s_R^\alpha \bar{d}_L^\beta s_R^\beta | \bar{K}^0 > \right| \simeq \frac{1.76 \times 10^{-3}\ \mathrm{GeV}^3}{M_T^2}.$$

From $\Delta M_K^{exp} \simeq 3.5 \times 10^{-15}$ GeV, we find the lower limit of M_T as

$$M_T \geq 7 \times 10^5\ \mathrm{GeV}.$$

This bound conflicts with the previous result

$$M_T < 47\ \mathrm{MeV} \quad (\text{for } \lambda < 1).$$

We do not consider the strongly interacting coupling, i.e., $\lambda \gg 1$.

$$(\ M_T = \sqrt{2\lambda}\ \tau_0\ )$$

FCNCs

In order to avoid the problem, we introduce a small dimensionless parameter ϵ to the timeon term

$$\bar{q}[i\gamma_5 \tau_0 F]q \to \bar{q}[i\gamma_5\ \epsilon\ \tau_0 F]q.$$

FCNC

$$M_T \geq 7 \times 10^5 \text{ GeV}$$

⇓

$$\frac{M_T}{\epsilon} \geq 7 \times 10^5 \text{ GeV}$$

Light quarks

$\tau_0 \simeq 33$ MeV since

$$m_u, m_d \propto \tau_0.$$

⇓

$$\epsilon\tau_0 \simeq 33 \text{ MeV}$$

If ϵ is very small, both the constraints from FCNC and light quark masses can be relaxed.

For example, if we assume $\tau_0 \sim M_T$ $(\lambda \sim 0.5)$, we obtain

$$M_T > 151 \text{ GeV}, \quad \epsilon < 0.22 \times 10^{-3}. \qquad \left(M_T = \sqrt{2\lambda}\ \tau_0\right)$$

FCNCs

By assuming the lowest value of M_T: $M_T = 151$ GeV and $\epsilon = 0.22 \times 10^{-3}$, we also calculate the mass parameters in B_s, B_d and D systems.

$$\Delta M_{B_s}^{timeon} \sim 0.365 \times 10^{-13}\ \text{GeV} \quad < \quad \Delta M_{B_s}^{exp} \simeq 1.17 \times 10^{-11}\ \text{GeV}$$

$$\Delta M_{B_d}^{timeon} \sim 0.178 \times 10^{-13}\ \text{GeV} \quad < \quad \Delta M_{B_d}^{exp} \simeq 3.34 \times 10^{-13}\ \text{GeV}$$

$$\Delta M_{D}^{timeon} \sim 0.205 \times 10^{-14}\ \text{GeV} \quad < \quad \Delta M_{D}^{exp} \simeq 1.40 \times 10^{-14}\ \text{GeV}$$

These results are sufficiently suppressed.

parameter	input	parameter	input
m_u	2.5×10^{-3} GeV	f_K	0.16 GeV
m_d	5×10^{-3} GeV	f_{B_s}	0.24 GeV
m_s	0.095 GeV	f_{B_d}	0.198 GeV
m_c	1.25 GeV	f_D	0.223 GeV
m_b	4.2 GeV	M_K	0.497 GeV
M_{B_s}	5.366 GeV	M_{B_d}	5.280 GeV
M_D	1.865 GeV		

[Kifune, Kubo and Lenz, *Phys. Rev.* D77, 076010 (2008)]

Origin of ϵ

Since the timeon is a gauge singlet scalar, it cannot be incorporated into the (renomalizable) SM.

$F_{ij}\overline{\psi_{Li}}\tau\psi_{Rj}$ is not invariant under the $SU(2)_L \times U(1)_Y$.

That is, the timeon model is an effective theory after EWSB.
If we consider a non-renomalizable interaction such as

$$\frac{F_{ij}}{\Lambda}\,\overline{\psi_{Li}}\tau\psi_{Rj}\Phi,$$

Φ : $SU(2)_L$ doublet scalar
Λ : typical enegy scale

the timeon term appears after Φ gets the VEV, $<\Phi>$.
In this case, $<\Phi>/\Lambda$ might be able to explain the existence of ϵ.

$$\frac{F_{ij}}{\Lambda}\,\overline{\psi_{Li}}\tau\psi_{Rj}\Phi \to \frac{<\Phi>}{\Lambda}F_{ij}\overline{\psi_{Li}}\tau\psi_{Rj} \equiv \epsilon F_{ij}\overline{\psi_{Li}}\tau\psi_{Rj} \quad ?$$

Lepton sector

Extension to the lepton sector

We extend the timeon model to the lepton sector

$$\mathcal{H}_l = \bar{\ell}_i [M_\ell + i\gamma_5 \, \epsilon \, \tau_0 F_l]_{ij} \ell_j + \bar{\nu}_i^{(c)} [M_\nu + i\gamma_5 \, \epsilon_\nu \, \tau_0 F_l]_{ij} \nu_j .$$

We put the same parameter as that of the quark sector.

$$\epsilon \simeq 0.22 \times 10^{-3}$$

$$(\ \tau_0 = 151 \text{ GeV} \ )$$

We introduce a new parameter as the origin of neutrino masses may be different from others, e.g., See-Saw mechanism.

Simple model

The matrix F_l takes the same form as that of the quark sector

$$F_l = \begin{pmatrix} \cos^2 \alpha_l & \sin\alpha_l \cos\alpha_l \cos\beta_l & \sin\alpha_l \cos\alpha_l \sin\beta_l \\ \sin\alpha_l \cos\alpha_l \cos\beta_l & \sin^2 \alpha_l \cos^2 \beta_l & \sin^2 \alpha_l \sin\beta_l \cos\beta_l \\ \sin\alpha_l \cos\alpha_l \sin\beta_l & \sin^2 \alpha_l \sin\beta_l \cos\beta_l & \sin^2 \alpha_l \sin^2 \beta_l \end{pmatrix},$$

but described by different angles: α_l and β_l.

Lepton sector

Simple model

$$\mathcal{H}_l = \bar{\ell}_i [M_\ell + i\gamma_5 \epsilon \tau_0 F_l]_{ij} \ell_j + \bar{\nu}_i^{(c)} [M_\nu + i\gamma_5 \epsilon_\nu \tau_0 F_l]_{ij} \nu_j .$$

We assume the following simple mass matrices.

$$\begin{pmatrix} b_l \eta_l^2 (1+\xi_l^2) & -b_l \eta_l & -b_l \xi_l \eta_l \\ -b_l \eta_l & b_l + a_l \xi_l^2 & -a_l \xi_l \\ -b_l \xi_l \eta_l & -a_l \xi_l & a_l + b_l \end{pmatrix}$$

$$l = \ell, \nu$$

$\eta_\ell = 0$
$\xi_\ell = -1$

$$M_\ell = \begin{pmatrix} 0 & 0 & 0 \\ 0 & b_\ell + a_\ell & a_\ell \\ 0 & a_\ell & a_\ell + b_\ell \end{pmatrix}$$

$\eta_\nu = -\sqrt{1/2}$
$\xi_\nu = 0$

$$M_\nu = \begin{pmatrix} \frac{1}{2} b_\nu & \sqrt{\frac{1}{2}} b_\nu & 0 \\ \sqrt{\frac{1}{2}} b_\nu & b_\nu & 0 \\ 0 & 0 & a_\nu + b_\nu \end{pmatrix}$$

These matrices lead to the exact tri-bi-maximal(TBM) mixing.

$$V_{TB} = \frac{1}{\sqrt{6}} \begin{pmatrix} 2 & \sqrt{2} & 0 \\ -1 & \sqrt{2} & -\sqrt{3} \\ -1 & \sqrt{2} & \sqrt{3} \end{pmatrix} \Longleftrightarrow \begin{array}{l} \sin^2\theta_{23} = 1/2 \\ \sin^2\theta_{12} = 1/3 \\ \sin^2\theta_{13} = 0 \end{array}$$

After the timeon gets the VEV, the mixing matrix deviates form TBM.

Lepton sector

Comments

- After the timeon gets the VEV, the mixing matrix deviates from the exact TBM pattern.

- $\alpha_l = 0$

$$F_l = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

⇒ No flavor changing timeon couplings in the charged lepton sector and $\sin\theta_{13} = 0$.

- $\alpha_l \neq 0$

Flavor changing timeon couplings in the charged lepton sector are induced, and it leads to $\sin\theta_{13} \neq 0$.

Lepton sector

Parameter space

The model has seven controllable parameters:

$$a_\ell,\ b_\ell,\ a_\nu,\ b_\nu,\ \alpha_l,\ \beta_l,\ \epsilon_\nu,$$

which can be fitted by seven physical quantities:

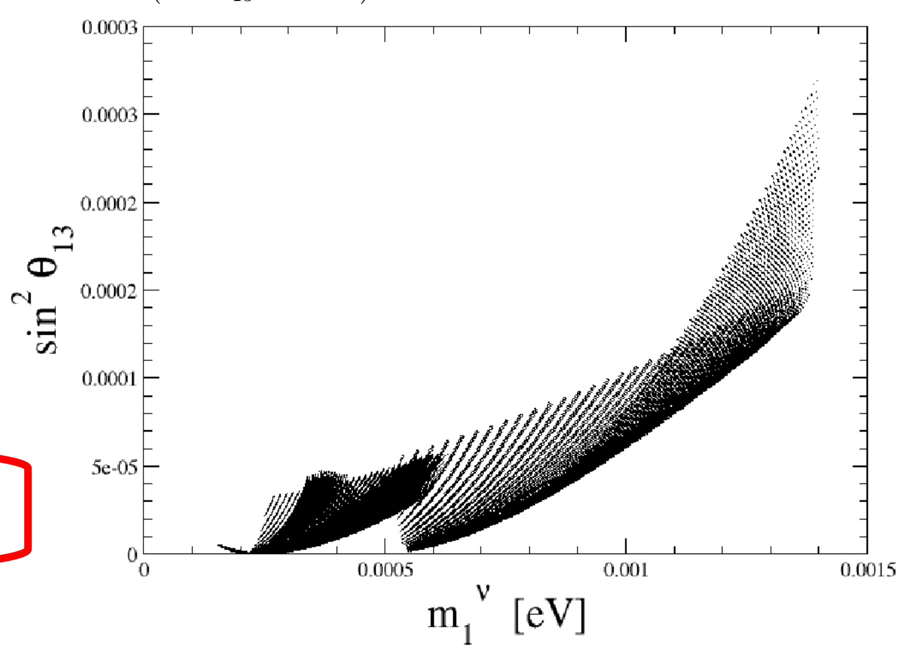
$$m_e = 0.511 \text{ MeV},\ m_\mu = 105.658 \text{ MeV},\ m_\tau = 1776.84 \pm 0.17 \text{ MeV},$$

$$\sin^2\theta_{12} = 0.288 \sim 0.326,\ \ \sin^2\theta_{23} = 0.44 \sim 0.57,$$

$$\Delta m_{21}^2 = (7.45 - 7.88) \times 10^{-5} \text{ eV}^2,$$

$$\Delta m_{31}^2 = (2.29 - 2.52) \times 10^{-3} \text{ eV}^2.$$

($\sin^2\theta_{13} < 0.026$)



From the right figure we find that
$\sin\theta_{13}$ must be small but $\sin\theta_{13} \neq 0$.

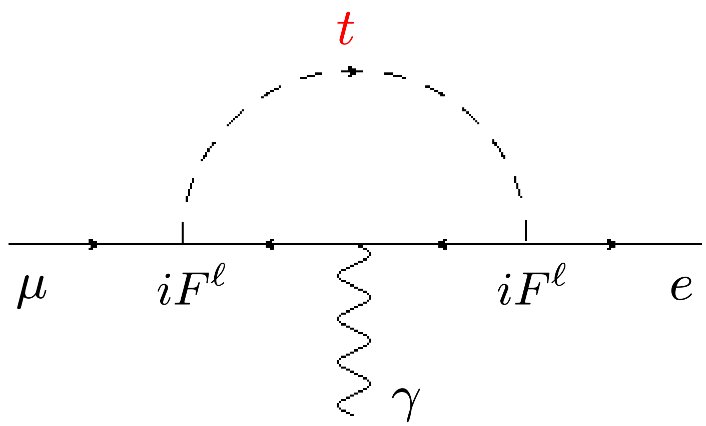
Non-zero flavor changing timeon couplings

Lepton sector

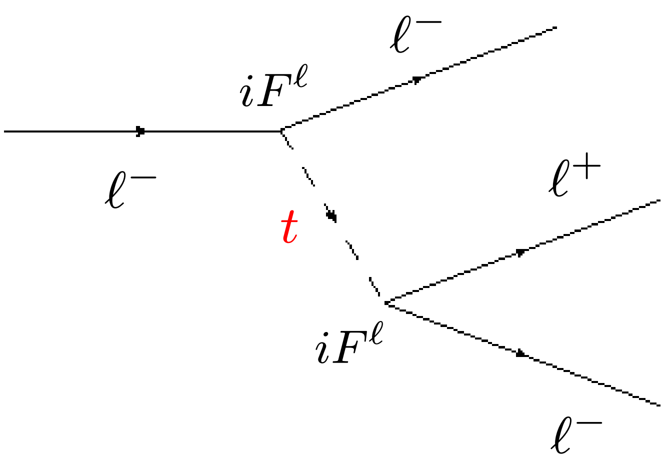
Leptonic processes

Now that, all the parameters included in the model are determined we can calculate some leptonic processes.

[$\mu \to e + \gamma$]



[$\ell^- \to \ell^- + \ell^+ + \ell^-$]



$$Br(\mu \to e\gamma) = \frac{\alpha_{em}\tau_\mu}{2^{10}\pi^4}\frac{m_\mu^3 m_\tau^2}{M_T^4}\left(\epsilon F^\ell_{\mu\tau}\ \epsilon F^\ell_{e\tau}\right)^2 \left|\ln\frac{m_\tau^2}{M_T^2} + \frac{3}{2}\right|^2$$

$$Br(\ell^- \to \ell_3^- \ell_2^+ \ell_1^-) = \frac{5}{3}\frac{\tau_\ell}{2^{11}\pi^3}\frac{m_\ell^5}{M_T^4}\left(\epsilon F^\ell_{\ell_3\ell}\ \epsilon F^\ell_{\ell_2\ell_1}\right)^2$$

Unfortunately, contributions form the timeon to these processes are too small to be measured in the near future.

Lepton sector

Leptonic processes

Now that, all the parameters included in the model are determined we can calculate some leptonic processes.

Lepton Flavor Violating decays

$$Br(\mu \to e\gamma) \simeq 10^{-16} \sim 10^{-21} \qquad Br^{exp}(\mu \to e\gamma) < 1.2 \times 10^{-11}$$

$$Br(\mu^- \to e^- e^+ e^-) \simeq 10^{-21} \sim 10^{-26} \qquad Br^{exp}(\mu^- \to e^- e^+ e^-) < 1.0 \times 10^{-12}$$

$$Br(\tau^- \to e^- e^+ e^-) \simeq 3 \times 10^{-21} \qquad Br^{exp}(\tau^- \to e^- e^+ e^-) < 3.6 \times 10^{-8}$$

$$Br(\tau^- \to \mu^- \mu^+ \mu^-) \simeq 10^{-19} \sim 10^{-35} \qquad Br^{exp}(\tau^- \to \mu^- \mu^+ \mu^-) < 3.2 \times 10^{-8}$$

Muon Anomalous Magnetic Moment

$$\Delta a_\mu \simeq 10^{-16} - 10^{-21} \qquad a_\mu^{exp} - a_\mu^{SM} \sim 10^{-11}$$

All results are consistent with the experimental data, but the contributions from the timeon are too small to be measured in the near future.

Summary

- We have investigated the timeon model.
- The original timeon model predicts a small timeon mass, but it conflicts with the constraints of FCNCs.
- We have introduced a small parameter ϵ and shown that the parameter can relax the constraints.
- We have found that $M_T > 151\ \mathrm{GeV}$ or $\epsilon < 0.22 \times 10^{-3}$.
- We have also extended the timeon model to the lepton sector and found that our model predicts non-zero flavor changing timeon couplings.
- Unfortunately, all contributions to the leptonic processes are not measurable in the near future.

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